1. Introduction

This is the third of papers concerned with the mechanism of confinement phase transition in quantum gluodynamics proposed recently by the author in collaboration with N. A. Sveshnikov [1, 2, 3]. The main result of those papers was realization that the conventional formulation of the Yang–Mills theory becomes incomplete on quantum level. Such drawback of the theory would be fatal if one wants to understand the origin of confinement.

Certainly what does confinement mean is a rather difficult question generally, but not so much at finite temperature. Obviously, it must imply singletness of the space of states we evaluate Gibbs average over

$$Z_s = \mathrm{Tr}_{\mathcal{H}_s}\, e^{-\beta H}\,, \qquad \mathcal{H}_s = P_s\,\mathcal{H}\,. \tag{1}$$

So the real question which should be asked is this. The singlet projector P_s of what group does stand in the formula above? The requirement of just global lack of colour turns out actually very weak and will not provide confinement. The Gauss law vanishing at some states has nothing in common with it at all, rather this is the way how physical states are defined in the enlarged phase space. The group of all local gauge transformations G is too large, and the global subgroup G_0 is too small to fit confinement. No doubts the truth lays somewhere between them. The explicit characterization of the confinement group can be constructed by the following manner. Let us denote by G_p the subgroup of the kind $G_p = \{g(\mathbf{x}) \in G : \ g(\mathbf{x}) \to 1,\ (|\,\mathbf{x}\,| \to \infty)\}$ of so called "small" or proper transformations, the quotient $G_\infty = G/G_p$ is then the group of purely "large" ones.

The group G_∞ of *gauge transformations at infinity* will be namely we are looking for. In any coordinate frame this group is represented by matrices depending on angles only $g(\hat{\mathbf{x}})$.

By virtue of the well–known Peter–Weyl theorem [4] the singlet projector (1) may be written down as an integral over Haar measure

$$P_s = \int_{G_\infty} \prod_{\hat{\mathbf{x}}} d\sigma(\hat{\mathbf{x}})\ e^{i\int d\hat{\mathbf{x}}\Phi(\hat{\mathbf{x}})\,\sigma(\hat{\mathbf{x}})}\,, \tag{2}$$

where Φ are the generators of G_∞.

Previous papers were devoted to that this formula could be understood as a consequence of dynamics of *variables at infinity.* Such a variables are in fact weak limits of the electric field integrals

$$\sigma(\hat{\mathbf{x}}) = w\text{-}\lim_{R\to\infty} \int_R^\infty dx\ \hat{\mathbf{x}}\mathbf{E}(\mathbf{x}) \tag{3}$$

along the ray from a point with radius R to infinity in $\hat{\mathbf{x}}$ direction. The accurate mathematical definition of variables at infinity requires applying of powerful algebraic QFT formalism enhanced by G. Morchio and F. Strocchi [5-7] to the case of long–range interactions.

We have shown in [1, 2] that for groups SU(2) and SU(3) projector (1) arrises after phase transition at temperatures below the critical $T < T_c$.

Quite reasonably one should investigate whether this effect has direct interplay to the law of static quarks interaction, because their linearly–rising potential usually associates in mind with the picture of confinement. The answer turns out positive as we shall see below.

Traditional approach to such task bases on Wilson loop evaluation [8-10]

$$\mathcal{W}[\Gamma] = \langle \mathrm{Tr\,P}\ \exp(g \oint_\Gamma A_\mu dx^\mu) \rangle, \quad A_\mu = A^a_\mu t^a, \quad (t^a)^\dagger = -t^a\,. \tag{4}$$

Indeed we must tackle a nonperturbative derivation that is the main problem of all. In the first, note that Wilson loop is invariant under gauge transformations and space translations, which are commuting between each other.

The serious advance may be accomplished in 3-dimensional Fock–Schwinger (FS) gauge [11-13]

$$\mathbf{x}\mathbf{A}(t, \mathbf{x}) = 0\,. \tag{5}$$

This very gauge is distinguished in two respects: the Gauss law constraint is exactly solvable in it and the residual gauge group coincides with the group G_∞ of *gauge transformations at infinity*. The generators of G_∞ in this gauge are

$$\Phi(\sigma) = \int d\hat{\mathbf{x}}\, \sigma^a(\hat{\mathbf{x}}) \int_0^\infty x^2\, dx\ \nabla \mathbf{E}^a_\perp(\mathbf{x}) = -\int d\hat{\mathbf{x}}\, \sigma^a(\hat{\mathbf{x}}) \lim_{R\to\infty} R^2 \hat{\mathbf{x}} \mathbf{E}^a(R\hat{\mathbf{x}})\,. \tag{6}$$

The former transformations are needless to be refixed completely, though this is also possible under the boundary condition $\mathbf{A}(t, \mathbf{0}) = 0$. The point is that the path integral is no longer divergent due to volume of gauge orbits because the path measure $\mathcal{D}\mathbf{A}$ is concentrated on the L^2 space. In the Coulomb gauge, say, the residual group contains an admixture of localized transformations that leads, as known, to Gribov copies problem. The case of FS gauge is remarkable since those bad transformations have been cancelled, and those which left are not dangerous. We may say even, that all things in FS gauge are sensible and physical enough, though evaluation of them is extremely tedious. The Wilson loop seems an exclusion of this rule as it evaluates in this gauge very easily. This occasion has of course good geometrical motivations just mentioned.

To utilize the gauge condition a special rectangular contour will be chosen. Since the integral (4) doesn't depend on the origin point in can be taken in such a way, that the spacial boundaries of the contour will be tending to infinity under a fixed distance between them.

The basic step in our derivation consists in correspondence between singlet projector and specification of the integration space for collective variables.

Confinement affects the theory through the vanishing condition

$$Q^a(\hat{\mathbf{x}})\,\mathcal{H} = 0\,, \qquad Q^a(\hat{\mathbf{x}}) = \int_0^\infty x^2\,dx\,\nabla\mathbf{E}^a_\perp(y\hat{\mathbf{x}}) \tag{7}$$

on the observable states. The latter condition being rewritten in terms of the radial Fourier–transform of the collective variable λ leads to a remarkable factorization property. All nontrivial quantum dynamics incorporates inside normalization constant apart from one auxiliary integral of quantum fluctuations. This circumstance induces to that quarks degrees of freedom themselves might be disdained without any trouble, so we shall omit them further.

The string tension in the area law is constituted by either ultraviolet and infrared divergences. Its particular form seems familiar in comparison with our previous results [1-3] on gluodynamics effective action. Namely, the ratio of the critical temperature to the squared root of the string tension

$$\xi \equiv T_c/\sqrt{\chi} \tag{8}$$

is a finite construction depending on the group dimension. Since χ presents but the only external parameter, the effective action being restricted on constant fields depends upon, one may even say about underlain string sense of the confined gauge theory.

An endeavour of analogical derivation in the deconfinement case appears to be out of immediate attainment. Our approach here bases heavily on the presence of P_s projector. If it would be extracted off an average, the latter ceases to refactorize itself and explicit evaluation becomes impossible as usually.

Though the possibility of the area law can not be completely ruled out at the deconfined phase, it seems very unlikely indeed. The perturbation theory predicts the perimeter law in such case, but we should not be truly content with a perturbative result.

All throughout the paper notations of [1, 2] are used.

2. Wilson loop

Let us consider the Wilson loop (4), that is a gauge–invariant order parameter. It is necessary to relate it with the physical variables of the Fock–Schwinger gauge to begin with. The zero component A_0 appears in the first order formalism as a Lagrange multiplier

$$\mathcal{L} = \dot{\mathbf{A}}\mathbf{E} - \frac{1}{2}((\mathbf{E})^2 + (\mathbf{B})^2) + A_0\nabla\mathbf{E}\,. \tag{9}$$

This variable is determined from the equation of motion

$$E^a_k = \partial_0 A^a_k - \partial_k A^a_0 + g t^{abc} A^b_k A^c_0 \tag{10}$$

while the task on conditional extremum is formulated. From the (10) we get after multiplication by $\hat{x}_k$

$$A_0(t, \mathbf{x}) = -\int_z^x dy\, E_\parallel(t, y\hat{\mathbf{x}})\,. \tag{11}$$

The lowest integral limit z is an arbitrary point, nevertheless such ambiguity disappears in the integral over closed circuit.

Otherwise it may be shown that for any functional F

$$\langle F[A_0(\mathbf{x})]\rangle = \langle F[\int_x^z dy\, E_\parallel(y\hat{\mathbf{x}})]\rangle\,. \tag{12}$$

Rewriting the average in the form $F[\frac{\delta}{\delta J_0}]\Big|_0 e^{i\,A_0(\Phi - J_0)}$, and performing integration over A_0 and $E_\parallel$ we really get

$$F[\frac{\delta}{\delta J_0}]\Big|_0 e^{\frac{1}{2}(E_\parallel - (\partial\hat{\mathbf{x}})^{-1} J_0)^2} = F[-(\frac{\partial}{\partial x})^{-1} E_\parallel]\, e^{\frac{1}{2}E_\parallel^2}\,. \tag{13}$$

The most suitable contour Γ consists of two straight lines along radius under $\hat{\mathbf{x}}$ direction at times $t = 0, L$, which are actually vanishing in virtue of gauge condition (5), and of two lines parallel to the time axis at radii $x = R', R''$. Hence there leaves from (4) just the contribution

$$\oint_\Gamma A_\mu dx^\mu = \int_0^L dt\,(A_0(t, R'\hat{\mathbf{x}}) - A_0(t, R''\hat{\mathbf{x}})\,) = -\int_0^L dt \int_{R'}^{R''} dx\, E_\parallel(t, \mathbf{x})\,. \tag{14}$$

To perform Gibbs averaging some more elaborated technique is needed. In [1, 3] there was proposed the method of the effective action denoted as W that is depending on collective variables $\lambda,\ \nu$. One may connect the generating functionals of longitudinal strengths

$$Z[\zeta, \eta] = \int_\leftrightarrow \mathcal{D}\mathbf{A}_\perp\, \mathcal{D}\mathbf{E}_\perp\ \exp{(-)} \int_\Omega dx\, [\,\mathbf{E}\dot{\mathbf{A}} - \frac{1}{2}\mathbf{E}_{FS}^2 + \frac{1}{2}\mathbf{B}_{FS}^2 + i\,\zeta E_\parallel - \eta B_\parallel\,] \tag{15}$$

and of collective fields ($\lambda = -\partial_x \sigma$)

$$\mathcal{Z}[\zeta, \eta] = \int_\leftrightarrow \mathcal{D}\,\lambda\ \mathcal{D}\,\nu\ \exp\,[\,-W[\lambda, \eta] - i\,(\,\lambda\,\zeta + \eta\,\nu\,)\,] \tag{16}$$

via the relation

$$Z[\zeta, \eta] = \exp\,[\,\frac{1}{2}\int_\Omega dx\ (\,\zeta^2 + \eta^2\,)\,]\ \mathcal{Z}[\zeta, \eta]\,. \tag{17}$$

Let $F(E_\parallel, B_\parallel)$ be an arbitrary functional of its arguments. Then its average

$$\langle\, F(E_\parallel, B_\parallel)\,\rangle = Z^{-1}[\zeta, \eta]\, F(\,i\,\frac{\delta}{\delta\,\zeta}\,,\,\frac{\delta}{\delta\,\eta}\,)\ Z[\zeta, \eta]\Big|_{\zeta=\eta=0} \tag{18}$$

is expressed through the $\langle F(\lambda,\nu)\rangle$ of $\mathcal{Z}[\zeta,\eta]$'s by means of the formula

$$\begin{array}{c}\langle\, F(\,E_{\|}\,,\,B_{\|}\,)\,\rangle = \int_{\leftrightarrow}\mathcal{D}\,\xi\;\mathcal{D}\,\theta\;\;\exp\,(\,-\tfrac{1}{2}\int_\Omega dx\,[\,\xi^2+\theta^2\,]\,)\\ \langle\, F(\,\lambda+i\,\xi\,,\;-i\,\nu+\theta\,)\,\rangle\,.\end{array} \tag{19}$$

Here integrations must be carried out over fields from L^2-space. This relation lies in a heart of collective variables interpretation. They ought be imagined as longitudinal (chromo) electric and magnetic fields smoothed by a Gaussian noise.

Combining previous results we infer the basic formula

$$\mathcal{W}[\Gamma] = \int_{\leftrightarrow}\mathcal{D}\xi\,\mathcal{D}\sigma\,\mathcal{D}\nu\,e^{-W[\sigma,\nu]-\frac{1}{2}\int_\Omega dx\,\xi^2}\;\mathrm{tr}\,\mathrm{T}\,\exp\left(ig\int_0^L dt\int_{R'}^{R''}dx\,[\lambda+i\,\xi](t,\mathbf{x})\right). \tag{20}$$

Time was converted into Euclidean ($L\le\beta$). Dyson $\mathrm{T}\exp$ stands because coefficients fields depend upon time. Trace with an upper–case letter means a trace of operators in the Hilbert space not commuting in different instants. A lower–case trace in (20) is a Lie–algebra adjoint representation one.

Confined phase is characterized by insertion of singlet projector (1) of the group of *gauge transformations at infinity* G_∞ into observables' Gibbs averages

$$\langle F\cdots G\rangle^{conf} = \langle P_s F\cdots G\rangle. \tag{21}$$

We should take this operator, which acts in the Hilbert space accepting a unitary representation of G_∞, into consideration when the auxiliary variable λ is introduced. It is important to stress that the integral over group can not be permuted with path integration. This ensues from noncommutativity of operators Q^a with different indices. Therefore we may evaluate the group integral in the end of calculations or include the projector as $P_s \propto \delta[Q]\equiv\prod_{\hat{\mathbf{x}},a}\delta[Q^a(\hat{\mathbf{x}})]$ via appropriate delta–function inside path integral. The second possibility is a more suitable here

$$\begin{aligned} Z &= \int_{\leftrightarrow}\mathcal{D}A_\perp\,\mathcal{D}E_\perp\,e^{-W[A,E]}\,\delta[Q] = \int_{\leftrightarrow}\mathcal{D}\lambda\,\mathcal{D}A_\perp\,\mathcal{D}E_\perp\,e^{\frac{1}{2}\int_\Omega dx\,(E_\perp^2-B^2)}\\ &\quad e^{\int_\Omega dx\,[-\frac{1}{2}\lambda^2+\lambda E_{\|}]}\,\delta[Q]\,. \end{aligned} \tag{22}$$

To be self–consistent we present our path integral in terms of product of usual integrals over Fourier–modes of the complete and orthonormal basis formed by the spherical waves

$$f_k(x) = \left(\frac{2}{\pi}\right)^{1/2}\frac{\sin\,kx}{kx} \tag{23}$$

multiplied by spherical functions Y_{lm}. Introducing new variable φ related with λ via

$$\lambda = \frac{1}{x^2}\int_0^x y^2\,dy\;\;\varphi(y\hat{\mathbf{x}})\,. \tag{24}$$

we express the integral of interest in the following manner

$$\mathcal{I} = \int \prod_{n,lm} d\tilde{\varphi}_{n,lm}(k)\; e^{\sum_{n,lm} \int_0^\infty dk \left[-\frac{1}{2} |\, \tilde{\varphi}\, |^2_{n,lm}(k) + \tilde{\Phi}^*_{n,lm}(k)\, \tilde{\varphi}_{n,lm}(k) \right]} . \qquad (25)$$

The confinement requirement implies then

$$\tilde{\Phi}_{n,lm}(k) \mid_{k=0} = 0 \,, \qquad (26)$$

that is equivalent up a normalization constant to taking integration over fields obeying $\tilde{\varphi}_{n,lm}(k=0) = 0$. The latter leads us to further simplifications.

Parameters R', R'' are in our disposition. We can choose them so that simplify evaluation a great deal. Namely let us arrange the limit of the kind $R', R'' \to \infty, \quad R'' - R' \equiv \Delta R = const$. In (20), the integral

$$\int_{R'}^{R''} dx\, \lambda_{n,\,lm}(x) \to -\frac{\Delta R}{R' R''} 2^{3/2} \pi^{1/2}\, \tilde{\varphi}_{n,\,lm}(0) = 0 \qquad (27)$$

tends to zero. So far we have to take just a simple integral over the auxiliary field ξ. For this purpose we first of all make the following remarks.

The generators of SU(N) fundamental representation are N dimensional matrices satisfying the conditions

$$\mathrm{tr}(\lambda^a\, \lambda^b) = 2\, \delta^{ab} \,, \qquad \mathrm{tr}\lambda^a = 0 \,. \qquad (28)$$

Coordinate frame would be conveniently to choose so that the matrices of $(N-1)$ commuting generators will be represented by diagonal matrices $\{\lambda^\alpha_{kk} \equiv \lambda^{(\alpha)}_k,\ (\alpha = 1, \ldots, N-1)\}$. Implementing gauge rotations all but $(N-1)$ of ξ's components in respect to these generators might be made zero:

$$\xi = \sum_{\alpha=1}^{N-1} \xi^\alpha\, \frac{\lambda^\alpha}{2\,i} \,. \qquad (29)$$

The characteristic condition (28) regains formally in terms of $\vec{\lambda}^{(\alpha)}$ N- dimensional vectors after introduction of the vector

$$\lambda^{(0)} = \frac{1}{\sqrt{N}} \big(\underbrace{1, \ldots, 1}_{N} \big) \,, \qquad (30)$$

as a requirement that the set $\left\{ \lambda^{(0)}, \lambda^{(\alpha)}/\sqrt{2} \right\}$ is an orthonormal frame in N-dimensional vector space. The completeness of such basis presupposes unit decomposition

$$\sum_{\alpha=1}^{N-1} \lambda^{(\alpha)}_i\, \lambda^{(\alpha)}_j = 2 \left(\delta_{ij} - \lambda^{(0)}_i\, \lambda^{(0)}_j \right) . \qquad (31)$$

These obvious remarks being done let us return to evaluation of our integral. In Appendix we prove that any normalized vector $\hat{\xi}(t)$ may be rotated to make it time–independent. Since then on the T-exponential becomes a usual one

$$J = \int_{\leftrightarrow} \mathcal{D}\vec{\xi}\, e^{-\frac{1}{2}\int d^4x\,\vec{\xi}^{\,2}} \,\mathrm{tr}\,\exp\left(-g \int_0^L dt \int_{R'}^{R''} dx\, \xi(t, x\hat{\mathbf{x}}) \right) . \qquad (32)$$

Provided special basis (29) choice the factorization of multiples takes place

$$J = \int_{\leftrightarrow} \prod_{\alpha=1}^{N-1} \mathcal{D}\xi^\alpha\, e^{-\frac{1}{2}\int d^4x\,(\xi^\alpha)^2} \sum_{i=1}^{N} \exp\left[\sum_{\alpha=1}^{N-1} (j \bullet \xi^\alpha)\, \lambda_i^{(\alpha)} \right] , \qquad (33)$$

$$j(x) = \frac{i\,g}{2}\,\theta(0 \le t \le L)\,\frac{\theta(R' \le x \le R'')}{x^2}\,\delta(\hat{\mathbf{x}} - \hat{\mathbf{x}}') \qquad (34)$$

giving rise to the result

$$J = \sum_{i=1}^{N} \exp\left[- \sum_{\alpha=1}^{N-1} (\lambda_i^{(\alpha)})^2\, \chi_{_{SU(N)}}\, S \right] \qquad (35)$$

and consequently,

$$\mathcal{W}_{SU(N)} = N\,\exp\left(-\chi_{_{SU(N)}}\, S \right) , \qquad S = L\,(R'' - R') \qquad (36)$$

with the string tension

$$\chi_{_{SU(N)}} = 2\left(1 - \frac{1}{N} \right) \chi\,. \qquad (37)$$

We have taken into account formula (31) for the sum inside the exponential. This expression is nothing but the long–awaited area law with the SU(2) string tension

$$\chi = \lim_{R', R'' \to \pi\,\delta(0)} \frac{g^2\,\delta(\hat{\mathbf{0}})}{R'\,R''} \,. \qquad (38)$$

Surely, the accurate definition of this limit needs some care. Both radii in a self–consistent derivation should be tended to the space infinity related with a more familiar infinity like $\delta(0)$ by

$$\delta(0) = \lim_{R\to\infty} \int_{-R}^{R} \frac{dx}{2\pi} \,. \qquad (39)$$

There are two proper way of understanding such infinities. First, everything being cut–offed and kept finite till the final result, as it conventionally was done, has no problems to cope with. Yet the chance to fight way through complicated expressions under retained cut–off is negligible. Next, infinite objects above

relat.	lattice	theoretical
$\frac{T_c^{SU(3)}}{T_c^{SU(2)}}$	1.16 ± 0.07	$\sqrt{3/2} \simeq 1.22$
$\frac{\xi^{SU(3)}}{\xi^{SU(2)}}$	1.04 ± 0.18	$3/(2\sqrt{2}) \simeq 1.06$

Table 1: Comparison of our theoretical predictions with lattice data.

arise naturally enough in nonstandard analysis [14]. Fortunately, we mustn't deepen in those dark mathematical areas since the construction (38) has been already familiar for us if reliable formula (39) is borne in mind. Comparison with critical temperature of [15-19] leads to the relation

$$\chi = \frac{\pi^2\, T_c^2}{a_c}\,, \qquad a_c \approx 2.61882\ldots \tag{40}$$

For the dimensionless ratio $\xi \equiv \frac{T_c}{\sqrt{\chi}}$ we predict in the case of SU(3) group the value

$$\xi_{SU(3)} = \frac{3}{2\sqrt{2}\,\pi}\sqrt{a_c} \approx 0.55\,, \tag{41}$$

whereas the lattice datum [15-19] is 0.58 ± 0.04. Another data are summarized in Tab. 1.

3. Appendix

We shall prove that for any normalized $\hat{\xi}(t)$ there always exists a solution of the following equation[1]

$$\frac{d}{dt}\Big(U^{-1}(t)\,\hat{\xi}(t)\,U(t)\Big) = 0\,, \tag{42}$$

that can be rewritten in terms of

$$[V(t)\,,\,\hat{\xi}(t)] = \partial_t\,\hat{\xi}(t)\,, \qquad V(t) = \partial_t\,U(t)\;U^{-1}(t)\,. \tag{43}$$

[1] The solution is needless to be unique.

As a consequence the sought for solution is the Dyson T-exponential

$$U(t) = \mathrm{T}\ \exp\left(\int^t d\tau\, V(\tau)\right)\ . \tag{44}$$

In a coordinate frame (43) becomes (N^2-1) dimensional matrix equation $(a,b,c = 1,\ldots,N^2-1)$

$$M^{a\,b}(\hat{\xi})\, V^b = \partial_t\, \hat{\xi}^a(t)\,, \qquad M^{a\,b}(\hat{\xi}) = t^{a\,b\,c}\, \hat{\xi}^c\,. \tag{45}$$

Since the matrix M is degenerate the solution will be not unique, so we shall try to build some of possible solutions. By appropriate extra transformation M can be presented in the form of direct sum[2]

$$M = \bigoplus_{\lambda=1}^{(N^2-N)/2} c_\lambda\, \mathcal{L}_\lambda\,, \tag{46}$$

where $\mathcal{L}_\lambda$, $\mathcal{P}_\lambda$ are operators with algebra

$$\begin{aligned} \mathcal{L}_\lambda\, \mathcal{L}_{\lambda'} &= -\delta_{\lambda\lambda'}\, \mathcal{P}_\lambda\,, \qquad \mathcal{P}_\lambda^2 = \mathcal{P}_\lambda\,, \\ \mathcal{L}_\lambda\, \mathcal{P}_\lambda &= \mathcal{P}_\lambda\, \mathcal{L}_\lambda = \mathcal{L}_\lambda\,. \end{aligned} \tag{47}$$

Then the general solution of (45) is

$$V = M^{-1}(\partial_t\, \hat{\xi}) + \tilde{V}\,, \quad M\, \tilde{V} = 0\,, \quad M^{-1} \equiv -\bigoplus_\lambda c_\lambda^{-1}\, \mathcal{L}_\lambda\,. \tag{48}$$

Really, one gets

$$\bigoplus_{\lambda'} c_{\lambda'}\, \mathcal{L}_{\lambda'} \left(-\bigoplus_{\lambda''} c_{\lambda''}^{-1}\, \mathcal{L}_{\lambda''}\ \partial_t\, \hat{\xi}\right) = \bigoplus_\lambda \mathcal{P}_\lambda\, \partial_t\, \hat{\xi} \equiv \mathcal{P}\, \partial_t\, \hat{\xi} = \partial_t\, \hat{\xi} \tag{49}$$

because $\mathcal{P}$ is the orthogonal projector onto the complement to the subspace $\{c\,\hat{\xi}\}$, that is orthogonal[3] to $\partial_t\,\hat{\xi}$:

$$\mathcal{P} = \mathbf{1} - \hat{\xi} \otimes \hat{\xi}\,, \qquad (\hat{\xi},\, \partial_t\, \hat{\xi}) = \mathbf{0}\,. \tag{50}$$

[2]To do this we can rotate $\hat{\xi}$ so that only $(N-1)$ components corresponding to commuting generators will be nonzero. Without loss of generality we imply that $\hat{\xi}$ has been already rotated in such a way.

[3]Indeed, matrix M's components with indices taking values in the set corresponding to $N-1$ nonzero components of $\hat{\xi}$ equal to zero due to complete antisymmetry of $t^{a\,b\,c}$.

4. Conclusion

This sufficiently simple paper build up a bridge between theoretical features of confinement and its interpretation.

From theoretical viewpoint the confinement phenomenon originates as a sequel of dynamical involvement of extra "classical" degrees of freedom taking place below the critical temperature. Certain it is the nature of this phase transition is essentially quantum and nonperturbative. Apparently the derivation that makes it possible to locate the effect under consideration was approximate though different from the perturbation expansion in terms of g. It was based on well suited for collective phenomena method of the effective action and the Hartree–Fock approximation in order to get the dynamics at infinity. Nevertheless the main result of our considerations seems undependable on the implications that lead to it, in a way it is an **explicit property of the theory**.

The visual picture of colour confinement following from the (21) is that *for any physical state contributing to the thermal average the flow of colour to infinity equals to zero in every direction.* Hence the coloured particles could not leak to spatial infinity and may no be observed so far.

Another possible insight on confinement is provided by Wilson loop. The matter of the present paper was to show how the first picture leads to the latter one. The results are not only positive but have also fair agreement with data of the lattice simulations [20-24].

We have unexpectedly disclosed that the combination of divergences constituting the only parameter of the gluodynamics effective potential possesses a direct physical sense. Such quantity is related with the string tension in the area law.

It is worthy to emphasize that the derivation of the area law was itself exact. The singletness under G_∞'s action has not been proved rigorously of course, though being taken for granted allows one to get such strong an assertion. The ratio ξ is known approximately as it depends on the critical temperature.

Certainly there are still a lot of subtleties and points to look at next. Analysis of the renormalization procedure is in any case indispensable. This will require accurate treatment of cut–offs. It is almost obvious that FS gauge is rather unsuitable for variety of calculations. Therefore to develop the systematic perturbation expansions for confined phase far more elaborated than naive is of immediate importance. The formula (1) prompts plausible ways to do so.

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