Partition function versus boundary conditions and confinement in the Yang-Mills theory

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We analyze the dependence of the partition function on the boundary condition for the longitudinal component of the electric field strength in gauge field theories. In a physical gauge the Gauss law constraint may be resolved explicitly expressing this component via an integral of the physical transversal variables. In particular, we study quantum electrodynamics with an external charge and SU(2) gluodynamics. We find that only a charge distribution slowly decreasing at spatial infinity can produce a nontrivial dependence in the Abelian theory. However, in gluodynamics for temperatures below some critical value the partition function acquires a delta-function-like dependence on the boundary condition, which leads to color confinement. [S0556-2821(98)02320-0]

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I. INTRODUCTION

There is considerable tradition in field theory (see, e.g., Ref. [1]) to neglect any surface terms inevitably appearing in derivations. This is usually motivated by a fast decrease of all fields at spatial infinity. Such behavior is only natural for theories with short-range interactions, but it is by no means obvious if long-range interactions come into play.

The same problem acquires a somewhat different form at finite temperature. Obviously, the partition function of a translationally invariant system is ill defined in infinite volume. Therefore, initially one has to enclose the system into a finite domain V and to assume some boundary conditions at the boundary ∂V. Then, to analyze dependence on boundary conditions we have to consider the functional,

$$Z[\chi]=\mathrm{Tr}[e^{-\beta H_V}\delta(\phi|_{\partial V}-\chi)], \tag{1}$$

where H_V is the Hamiltonian of the system in volume V, β is the inverse temperature, ϕ is some subset of the canonical variables, and the function χ defined on the boundary ∂V specifies the Dirichlet boundary conditions for the latter. In this framework we shall call $Z[\chi]$ the *effective partition function*, its introduction being analogous in spirit to that of the effective action. It is reasonable to expect that $Z[\chi]$ and any thermodynamic function become independent of the particular choice of boundary conditions in the thermodynamic limit $V\to\infty$. Once again, one may doubt whether that would really be the case for long-range interacting systems.

It was early recognized in the theory of gravity [2] that there are important physical situations in which boundary terms may have a physical meaning. We would like to mention also that many problems in hydrodynamics, such as, e.g., the description of surface waves [3], do require one to consider variables at the boundary and nonvanishing surface terms.

Recently interest in boundary effects in various field theories has rapidly increased [4]. It has been found in the framework of the algebraic quantum field theory [5] that certain two-dimensional models possess a nontrivial dynamics of the *variables at infinity*, and that such dynamics is responsible for the phenomenon of dynamic mass generation.

Our current purpose is to emphasize the role of boundary terms in the four-dimensional gauge field theory at finite temperature and to study the physical effects they can produce. In this case the appearance of a nontrivial χ dependence in $Z[\chi]$ in the thermodynamic limit is almost obvious for the following reasons. The gauge theory is initially formulated in terms of an enlarged set of variables, the vector potentials $\boldsymbol{A}$ and the electric field strengths $\boldsymbol{E}$ in the Hamiltonian formulation, which make the gauge invariance explicitly manifest. Further, to obtain observable quantities one has to project the theory onto a subset of the *physical* variables by resolving the Gauss law constraint $\nabla\mathbf{E}=\rho$ and by adopting a gauge condition. Boundary conditions, of course, have to be compatible with these. For instance, in the Abelian case consider the boundary condition for the electric field strength component $E_\parallel$ normal to the boundary ∂V, which we take for simplicity as a sphere of radius R:

$$R^2E_\parallel(R\hat{\boldsymbol{x}})|_{\partial V}=\chi(\hat{\boldsymbol{x}}). \tag{2}$$

This variable then has to obey the integrated form of the Gauss law and, therefore,

$$Z[\chi]\propto\delta\left(\int_{\partial V}d\hat{\boldsymbol{x}}\,\chi-\int_V d\boldsymbol{x}\,\rho\right). \tag{3}$$

We would like to emphasize that the dependence of the effective partition function on the boundary condition imposed on $E_\parallel$ is of primary importance in the gauge theory because of the direct relation of this component to the color charge flux due to the Gauss law. Analysis of this dependence will be the subject of the current paper. Based on knowledge of this dependence alone we can suggest a simple *confinement criterion*

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$$Z[\chi]\propto\prod_{\hat{x}}\ \delta(\chi(\hat{\boldsymbol{x}})). \tag{4}$$

The latter condition simply means that the color flux is strictly zero in *every* direction at spatial infinity for any state belonging to the Hilbert space of the system.

The plan of the paper is as follows. In Sec. II we proceed with a careful calculation of of $Z[\chi]$ in the simplest case of the Abelian theory with an external charge density. Then, in Sec. III we reproduce the answer obtained in the previous section using another technique, which is also applicable to the non-Abelian theory. Section IV is devoted to the calculation of $Z[\chi]$ using the mean-field approximation and to a consequent analysis of the confinement phase transition in gluodynamics.

II. PARTITION FUNCTION OF QED WITH AN EXTERNAL CHARGE

The partition function in a finite domain V may be represented by a Euclidean path integral over fields periodic in time on the interval $[0,\beta]$, where β is the inverse temperature. The path integral is well defined only if some boundary conditions are specified on all fields at the boundary ∂V.

Let us denote the transversal and longitudinal components of vectors with respect to the gradient $\boldsymbol{\partial}$ as superscripts $\boldsymbol{\partial A}^{\perp}=0$ and with respect to the vector $\boldsymbol{x}$ as subscripts $\boldsymbol{x}\mathbf{A}_{\perp}=0$. For simplicity we choose the Coulomb gauge $\boldsymbol{A}=\boldsymbol{A}^{\perp}$. We note also that the condition $\boldsymbol{A}=\boldsymbol{A}_{\perp}$ corresponds to the Fock-Schwinger gauge [6] (see the Appendix). We shall also assume that $V=\{\boldsymbol{x}:|\boldsymbol{x}|=R\}$ is a spherical domain with the radius R. The Gauss law constraint

$$\boldsymbol{\partial E}=\rho(\boldsymbol{x}) \tag{5}$$

allows us to eliminate one space component of the electric field strength $\boldsymbol{E}$.

In a previous work [7] on the basis of a general result [8] we have developed a Hamiltonian formalism for a system in a finite spherical domain incorporating the boundary values as Hamiltonian variables. We have shown that the boundary conditions $\boldsymbol{E}_{\perp}(R\hat{\boldsymbol{x}})=0$ and $x_jF_{ij}(R\hat{\boldsymbol{x}})=0$ are consistent with the localized time evolution in the Fock-Schwinger gauge. By transforming the theory into the Coulomb gauge one would arrive instead at a boundary condition of the form $\boldsymbol{E}^{\perp}(R\hat{\boldsymbol{x}})=0$. Since these variables are independent, the dependence on a particular choice of boundary conditions disappears for an infinite system. The situation is quite different for the component $E_{\parallel}(R\hat{\boldsymbol{x}})=\hat{\boldsymbol{x}}\boldsymbol{E}(R\hat{\boldsymbol{x}})$. Indeed, Eq. (5) may be easily solved:

$$E_{\parallel}(\boldsymbol{x})=\frac{1}{x^2}\int_0^x y^2dy(\rho-\boldsymbol{\partial E}_{\perp})(y\hat{\boldsymbol{x}}). \tag{6}$$

Both types of transversal variables are connected by the relation [9]

$$\boldsymbol{E}_{\perp}(\boldsymbol{x})=\boldsymbol{E}^{\perp}(\boldsymbol{x})-\boldsymbol{\partial}\int_0^x dy\ \hat{\boldsymbol{x}}\boldsymbol{E}^{\perp}(y\hat{\boldsymbol{x}}). \tag{7}$$

Combination of Eqs. (6) and (7) now yields

$$R^2E_{\parallel}(R\hat{\boldsymbol{x}})-\int_0^R y^2dy\ \rho(y\hat{\boldsymbol{x}})=\hat{\Delta}\int_0^R(R-y)dy\hat{\boldsymbol{x}}\boldsymbol{E}^{\perp}(y\hat{\boldsymbol{x}}), \tag{8}$$

where we have used the spherical part of the Laplacian $\Delta\equiv x^{-1}\partial_x^2x+x^{-2}\hat{\Delta}$. It is clear that the requirement $E_{\parallel}(R\hat{\boldsymbol{x}})=\chi(\hat{\boldsymbol{x}})$, where $\chi(\hat{\boldsymbol{x}})$ is arbitrary, is nothing but a constraint on the physical variables $\boldsymbol{E}^{\perp}$. As we have seen [7], it is this constraint that makes the finite volume Hamiltonian formalism closed.

Now, then, the partition function of QED with an external charge may be represented by the following path integral in the Coulomb gauge:

$$\begin{aligned}Z=\int \mathcal{D}\boldsymbol{A}\mathcal{D}\boldsymbol{E}\,\delta(\boldsymbol{\partial A})\,\delta(R^2E_{\parallel}(R\hat{\boldsymbol{x}})-\chi(\hat{\boldsymbol{x}}))\\ \times\exp\int_{\Lambda}d^4x\Bigg(i\boldsymbol{E}\dot{\boldsymbol{A}}-\frac{1}{2}\boldsymbol{E}^2+\frac{1}{2}\boldsymbol{A}\Delta\boldsymbol{A}\\ -\frac{1}{2\epsilon^2}(\boldsymbol{\partial E}-\rho)^2\Bigg),\end{aligned} \tag{9}$$

where we have used the notation for the domain $\Lambda=[0,\beta]\times V$ (we shall also use the notation $\partial\Lambda=[0,\beta]\times\partial V$). We have also introduced a regularization of the Gauss law by the parameter ϵ, which should be set equal to zero at the end of calculations. This is intended simply to avoid the use of singular distributions during intermediate derivations. As ϵ vanishes the ''soft'' Gauss law becomes the strict one in analogy with the one-dimensional formula

$$\lim_{\epsilon\to0}(2\pi\epsilon^2)^{-1/2}\exp(-\Phi^2/2\epsilon^2)=\delta(\Phi),$$

and the gauge invariance is completely restored.

The above Gaussian integral is evaluated in a standard manner by a shift of the integration variables. To find the integral over $\boldsymbol{E}$ we introduce a new integration variable $\boldsymbol{E}_1$:

$$\boldsymbol{E}=\boldsymbol{E}_1+\boldsymbol{\mathcal{E}},\quad \boldsymbol{E}_1(R\hat{\boldsymbol{x}})=0. \tag{10}$$

Here the new variable $\boldsymbol{E}_1$ satisfies the zero boundary condition and $\boldsymbol{\mathcal{E}}$ is chosen so that there is no linear term in $\boldsymbol{E}_1$. This gives an equation on $\boldsymbol{\mathcal{E}}$:

$$i\dot{\boldsymbol{A}}-\boldsymbol{\mathcal{E}}+\frac{1}{\epsilon^2}\boldsymbol{\partial}(\boldsymbol{\partial\mathcal{E}})-\frac{1}{\epsilon^2}\boldsymbol{\partial}\rho=0, \tag{11}$$

$$R^2\hat{\boldsymbol{x}}\boldsymbol{\mathcal{E}}(R\hat{\boldsymbol{x}})=\chi(\hat{\boldsymbol{x}}). \tag{12}$$

The latter boundary condition follows from the second delta function in Eq. (9). We may decompose this vector into transversal and longitudinal parts in momentum space, $\boldsymbol{\mathcal{E}} = \boldsymbol{\mathcal{E}}^{\perp} - \boldsymbol{\partial}\varphi$. Then, the transversal part is simply $\boldsymbol{\mathcal{E}}^{\perp} = i\dot{\boldsymbol{A}}$, and the equation for φ becomes

$$(\Delta - \epsilon^2)\varphi = -\rho, \tag{13}$$

$$R^2 \frac{\partial\varphi}{\partial R} = -\chi(\hat{\boldsymbol{x}}). \tag{14}$$

The partition function (9) is further decomposed as the product:

$$Z = Z_1 \tilde{Z},$$

$$Z_1 = \int \mathcal{D}\boldsymbol{A}^{\perp}\mathcal{D}\boldsymbol{E} \exp \int_{\Lambda} d^4x \times \left(-\frac{1}{2}\dot{\boldsymbol{A}}^2 + \frac{1}{2}\boldsymbol{A}\Delta\boldsymbol{A} - \frac{1}{2}\boldsymbol{E}_1^2 - \frac{1}{2\epsilon^2}(\boldsymbol{\partial E}_1)^2 \right), \tag{15}$$

$$\tilde{Z} = \exp \beta \left(\frac{1}{2}\int_{\partial V} d\hat{\boldsymbol{x}}\, \chi(\hat{\boldsymbol{x}})\varphi(R\hat{\boldsymbol{x}}) - \frac{1}{2}\int_{\partial V} d\boldsymbol{x}\, \rho(\boldsymbol{x})\varphi(\boldsymbol{x}) \right), \tag{16}$$

where φ is the solution of Eqs. (13),(14).

The solution of Eqs. (13),(14) is, obviously, the sum of the homogeneous part ϕ satisfying the nontrivial boundary condition and of the inhomogeneous part satisfying the zero boundary condition,

$$\varphi = \phi - G \bullet \rho, \quad G = (\Delta - \epsilon^2)^{-1}, \tag{17}$$

where G is the Green function corresponding to the zero-Neumann boundary condition at R. The effective partition function can be presented as

$$\tilde{Z} = \tilde{Z}_{\chi}\tilde{Z}_{\rho\rho}\tilde{Z}_{\rho\chi},$$

$$\tilde{Z}_{\chi} = \exp\left(\frac{\beta}{2}\int_{\partial V} d\hat{\boldsymbol{x}}\, \chi(\hat{\boldsymbol{x}})\phi(\hat{\boldsymbol{x}}) \right), \tag{18}$$

$$\tilde{Z}_{\rho\rho} = \exp\left(\frac{\beta}{2}\int_{\partial V} d\boldsymbol{x} d\boldsymbol{y} \rho(\boldsymbol{x}) G(\boldsymbol{x},\boldsymbol{y})\rho(\boldsymbol{y}) \right), \tag{19}$$

$$\tilde{Z}_{\rho\chi} = \exp\left(-\frac{\beta}{2}\int_{\partial V} d\boldsymbol{x}\, \phi(\boldsymbol{x})\rho(\boldsymbol{x}) \right) \times \exp\left(-\frac{\beta}{2}\int_{\partial V} d\hat{\boldsymbol{x}}\, \chi(\hat{\boldsymbol{x}})(G\bullet\rho)(\hat{\boldsymbol{x}}) \right). \tag{20}$$

It is natural to consider the problem further in terms of the spherical coordinates. The solution of Eq. (13), regular inside the sphere, is given by

$$\phi_{lm} = C_{lm}\sqrt{\frac{\pi}{2\epsilon r}} I_{l+1/2}(\epsilon r), \tag{21}$$

$$I_{n-1/2}(z) = \sqrt{\frac{2}{\pi z}} z^n \left(\frac{1}{z}\frac{d}{dz} \right)^n \cosh z. \tag{22}$$

The constant C_{lm} is determined from Eq. (14). The role of the regulator ϵ now becomes clear. The zero mode solution is, simply,

$$\phi_{00} = C_{00}\frac{\sinh \epsilon r}{\epsilon r},$$

$$C_{00} = -\frac{\chi_{00}}{R^2 \epsilon[\cosh\epsilon R/\epsilon R - \sinh\epsilon R/(\epsilon R)^2]}, \tag{23}$$

and it is $1/\epsilon^2$ singular as ϵ tends to zero. At the same time, solutions for other modes are perfectly regular in this limit and tend to

$$\phi_{lm} = C_{lm} r^l, \quad C_{lm} = -\frac{\chi_{lm}}{lR^{l+1}}. \tag{24}$$

In consideration of the zero mode one must therefore be more careful and keep ϵ nonvanishing. The zero-mode Green function, which is defined by

$$\left(\frac{1}{r}\frac{\partial^2}{\partial r^2} r - \epsilon^2 \right) G_{00}(r,r') = \frac{\delta(r-r')}{rr'}, \tag{25}$$

$$\frac{\partial G_{00}(r,r')}{\partial r}\Big|_{r=R} = 0, \tag{26}$$

is easily calculated:

$$G_{00}(r,r') = \frac{1}{\epsilon r r'}\left(\frac{1}{2}\sinh\epsilon|r-r'| - \frac{1}{2}\sinh\epsilon(r+r') + \frac{\sinh\epsilon R - \cosh\epsilon R/\epsilon R}{\cosh\epsilon R - \sinh\epsilon R/\epsilon R}\sinh\epsilon r \sinh\epsilon r' \right). \tag{27}$$

The leading terms at small ϵ are

$$G_{00}(r,r') \simeq -\frac{3}{\epsilon^2 R^3} + \frac{9}{5R} - \frac{1}{2}\frac{r^2 + r'^2}{R^3} - \frac{1}{\max(r,r')}, \tag{28}$$

$$\phi_{00} \simeq -\left(\frac{3}{\epsilon^2 R^3} - \frac{3}{10R} + \frac{r^2}{2R^3} \right)\chi_{00}. \tag{29}$$

Substitution of these results into formulas (18)–(20) gives, for the nonzero modes in the limit $\epsilon = 0$,

$$\tilde{Z}_{\chi l>0} = \exp\left(-\frac{\beta}{2R}\sum_{lm>0}\frac{|\chi_{lm}|^2}{l} \right). \tag{30}$$

For simplicity, we assume that the distribution of the charge ρ is spherically symmetrical. In this case only the zero-mode term survives. If we introduce the charge density momenta

$$\mathcal{Q}_{00}=\int_0^R r^2 dr\,\rho_{00}(r),\quad \mathcal{G}_{00}=\int_0^R r^4 dr\,\rho_{00}(r), \tag{31}$$

our results may be summarized as follows:

$$\begin{aligned}\tilde{Z}_{00}=\exp\beta\Bigg[&-\frac{1}{2}\int_0^R x^2dx\,y^2dy\,\frac{\rho_{00}(x)\rho_{00}(y)}{\max(x,y)}\\&-\frac{3}{2\epsilon^2R^3}(\mathcal{Q}_{00}-\chi_{00})^2\\&+\frac{1}{R}\left(\frac{9}{10}\mathcal{Q}_{00}^2-\frac{1}{10}\chi_{00}^2-\frac{3}{10}\chi_{00}\mathcal{Q}_{00}\right)\\&-\frac{1}{2R^3}(\mathcal{Q}_{00}-\chi_{00})\mathcal{G}_{00}\Bigg].\end{aligned} \tag{32}$$

In the limit $\epsilon\to 0$ this functional contains the delta function of the condition $\mathcal{Q}_{00}=\chi_{00}$, and in addition we find the following correction to the standard answer due to the surface terms:

$$\tilde{Z}_{00}=\exp\left(\frac{\beta}{2R}\mathcal{Q}_{00}^2\right)\delta(\mathcal{Q}_{00}-\chi_{00}). \tag{33}$$

As a simple illustration of the above result it is instructive to consider the charge density $\rho_{00}=-\kappa/(\sqrt{\pi}r)$ corresponding to a linear rising electric potential $\varphi=\kappa r$. For such an exotic charge distribution, modeling the confinement like potential, we find an additional constant contribution to the free energy density:

$$\Delta\mathcal{F}=-\frac{\log Z}{\beta V}=-\frac{3}{32\pi^2}\kappa^2. \tag{34}$$

It is interesting to note that this correction makes the free energy density smaller, and in this sense the boundary effects are thermodynamically significant. This example exhibits a promising connection between boundary effects and the confinement phenomenon.

III. COLLECTIVE VARIABLE FORMULATION

In the present section we give a different formulation of the same problem by introducing the collective variable σ, conjugate to the Gauss law constraint. Both formulations are completely equivalent in the Abelian theory, the transformation between them being just a trivial change of variables. However, the new formulation appears to be more fruitful in the non-Abelian gauge theory. Let us rewrite formula (9) in terms of the collective variable σ, introduced by the definition

$$\begin{aligned}&\exp\left(-\frac{1}{2\epsilon^2}\int_\Lambda d^4x(\boldsymbol{\partial E}-\rho)^2\right)\\&\quad=\int\mathcal{D}\sigma\exp\int_\Lambda d^4x\left(-\frac{\epsilon^2}{2}\sigma^2+i\sigma(\boldsymbol{\partial E}-\rho)\right).\end{aligned} \tag{35}$$

One starts by taking the integral over $\boldsymbol{E}$:

$$\begin{aligned}J=\int\mathcal{D}\boldsymbol{E}\exp\int_\Lambda d^4x\left(-\frac{1}{2}\boldsymbol{E}^2+i\boldsymbol{E}\dot{\boldsymbol{A}}+i\sigma\boldsymbol{\partial E}\right)\\ \times\delta(R^2E_\parallel(R\hat{\boldsymbol{x}})-\chi(\hat{\boldsymbol{x}})).\end{aligned} \tag{36}$$

This can be done in analogy with the previous section by applying a shift $\boldsymbol{E}=\boldsymbol{E}_1+\boldsymbol{\mathcal{E}}$ and using the decomposition $\boldsymbol{\mathcal{E}}=\boldsymbol{\mathcal{E}}^\perp-\boldsymbol{\partial}\varphi$. Then, we shall get $\boldsymbol{\mathcal{E}}^\perp=i\dot{\boldsymbol{A}}$ and $\varphi=i\sigma$. Using the delta function in the above formula one may rewrite the boundary term as $i\int_{\partial\Lambda}dt\,d\hat{\boldsymbol{x}}\,\chi\sigma$. In the term $\int d\boldsymbol{x}\,\dot{\boldsymbol{A}}\boldsymbol{\partial}\sigma$ the appropriate boundary term vanishes by the gauge condition. Thus, we obtain

$$\begin{aligned}J=\exp\left(-\frac{1}{2}\int_\Lambda d^4x[\dot{\boldsymbol{A}}^2+(\boldsymbol{\partial}\sigma)^2]+i\int_{\partial\Lambda}dt\,d\hat{\boldsymbol{x}}\,\chi\sigma\right)\\ \times\delta\left(iR^2\frac{\partial\sigma}{\partial R}+\chi\right).\end{aligned} \tag{37}$$

At this stage the χ dependence is contained only in the integral:

$$\begin{aligned}\tilde{Z}=\int\mathcal{D}\sigma\exp\Bigg[-\int_\Lambda d^4x\left(\frac{1}{2}(\boldsymbol{\partial}\sigma)^2+\frac{\epsilon^2}{2}\sigma^2+i\sigma\rho\right)\\+i\int_{\partial\Lambda}dt\,d\hat{\boldsymbol{x}}\chi\sigma\Bigg]\delta\left(iR^2\frac{\partial\sigma}{\partial R}+\chi\right).\end{aligned} \tag{38}$$

Such an integral is evaluated straightforwardly by a shift of the integration variable, $\sigma=\sigma_1+\varsigma$. The new variable satisfies the trivial boundary condition $R^2\partial\sigma_1(R\hat{\boldsymbol{x}})/\partial R=0$, and ς is found by the requirement that there be no term linear in σ_1 after the shift. This gives the equations on ς:

$$(\Delta-\epsilon^2)\varsigma=i\rho,\quad R^2\frac{\partial\varsigma}{\partial R}=i\chi. \tag{39}$$

Using the boundary condition on ς we finally derive

$$\tilde{Z}=\exp\left(-\frac{i}{2}\beta\int_V d\boldsymbol{x}\,\varsigma\rho+\frac{i}{2}\beta\int_V d\hat{\boldsymbol{x}}\,\varsigma\chi\right). \tag{40}$$

Note that with the identification $\varsigma=-i\varphi$ we have precisely reproduced the answer of Eqs. (13),(14),(16).

IV. NON-ABELIAN THEORY

In a previous paper [9] we have derived a representation for the partition function Z of SU(N) gluodynamics in a finite volume as the path integral over the collective variables analogous to that of the previous section. In order to find the effective partition function $Z[\chi]$ dependence we have to return to the beginning of that derivation in the Fock-Schwinger gauge:

$$Z[\chi]=\int \mathcal{D}\boldsymbol{A}\,\mathcal{D}\boldsymbol{E}\,\mathcal{D}\sigma \times\exp\left[\int_\Lambda d^4x\left(i\boldsymbol{E}\dot{\boldsymbol{A}}-\frac{1}{2}\boldsymbol{E}^2-\frac{1}{2}\boldsymbol{B}^2+i\sigma\nabla\boldsymbol{E}\right)\right] \times\delta(A_\parallel)\,\delta(R^2E_\parallel(R\hat{\boldsymbol{x}})-\chi), \tag{41}$$

where σ, clearly, is just a different notation for the temporal component of the gauge field, A_0. Here and below we shall suppress color indices if it does not lead to confusion. In the non-Abelian theory the gauge invariance of $Z[\chi]$ so defined is by no means obvious. We do not address this question here, restricting our considerations to the Fock-Schwinger gauge only. However, we shall see that the final expressions for the mean values of observables turn out gauge invariant. Obviously, any dependence on χ is concentrated in the path integral over $E_\parallel$:

$$I=\int \mathcal{D}E_\parallel\exp\left[\int_\Lambda d^4x\left(-\frac{1}{2}E_\parallel^2+i\sigma(\boldsymbol{\partial}\hat{\boldsymbol{x}})E_\parallel\right)\right]\delta(R^2E_\parallel-\chi). \tag{42}$$

This is calculated by a shift $E_\parallel=E_\parallel^1+\mathcal{E}$, where $\mathcal{E}=-i\partial\sigma/\partial x$ and $E_\parallel^1$ satisfies the zero Dirichlet-boundary condition. Such a derivation gives

$$I=\exp\left\{\int_0^\beta dt\left[-\frac{1}{2}\int_V d\boldsymbol{x}\left(\frac{\partial\sigma}{\partial x}\right)^2+i\int d\hat{\boldsymbol{x}}\sigma\chi\right]\right\} \times\delta\left(R^2\frac{\partial\sigma}{\partial R}-i\chi\right). \tag{43}$$

Next, by introducing the path integral representation

$$\exp\left(-\frac{1}{2}\int_\Lambda d^4xB_\parallel^2\right)=\int \mathcal{D}\nu\,\exp\left[\int_\Lambda d^4x\left(-\frac{1}{2}\nu^2+i\nu B_\parallel\right)\right], \tag{44}$$

and after performing the integrations over $\boldsymbol{A}_\perp$ and $\boldsymbol{E}_\perp$ (see Ref. [9] for more details) we finally obtain

$$Z[\chi]=\int \mathcal{D}\sigma\,\mathcal{D}\nu\,\exp\left(-W[\sigma,\nu]+i\int_{\partial\Lambda}dt\,d\hat{\boldsymbol{x}}\,\sigma\chi\right) \times\delta(R^2\sigma'-i\chi),$$

$$2W[\sigma,\nu]=\nu\bullet\nu+\boldsymbol{\partial}\sigma\bullet\boldsymbol{\partial}\sigma+\boldsymbol{K}_-\bullet C_+^{-1}\bullet\boldsymbol{K}_+ +\boldsymbol{K}_+\bullet C_-^{-1}\bullet\boldsymbol{K}_-+\mathrm{tr}\ln C_+C_-,$$

$$C_\pm=-\Delta_x-\nabla_t^2\pm D,\quad \boldsymbol{K}_\pm=\boldsymbol{\partial}_\pm\nu\pm\nabla_t\boldsymbol{\partial}_\pm\sigma,$$

$$\nabla_t^{ab}=\delta^{ab}\partial_t-gt^{abc}\sigma^c,\quad D^{ab}=gt^{abc}\nu^c, \tag{45}$$

where the projected derivatives are defined by

$$\partial_\pm^i=\Pi_\pm^{ij}\partial_j,\quad \Pi_\pm^{ij}=\frac{1}{2}(\delta^{ij}-\hat{x}^i\hat{x}^j\pm i\epsilon^{ijk}\hat{x}^k), \tag{46}$$

and the bullet denotes 4D integration over the domain Λ.

The effective action $W[\sigma,\nu]$ is rather complicated. The standard statistical mechanical procedure would be to start by considering the mean-field approximation in terms of these collective variables. Such an approximation is nontrivial compared to that in terms of the original gauge fields since we have included certain quantum fluctuations by exactly integrating over the gauge fields. Moreover, since the new variables transform homogeneously, one avoids concerns about breaking the gauge invariance of the theory while using simple stationary point solutions. In the Conclusion we shall also discuss mathematical justification for the current mean-field expansion based on the observation that the resulting effective action turns out to be proportional to a large parameter.

In the saddle point approximation one expands the action near the saddle point:

$$W[\varsigma+\sigma_1]=W[\varsigma]+\int_\Lambda d^4x\frac{\delta W}{\delta\varsigma(\boldsymbol{x})}\sigma_1(\boldsymbol{x}) +\int_{\partial\Lambda}dt\,R^2d\hat{\boldsymbol{x}}\mathcal{E}^{(1)}[\varsigma]\sigma_1(R\hat{\boldsymbol{x}})+\cdots. \tag{47}$$

In the mean-field approximation we may write

$$\tilde{Z}[\chi]=\exp\left(-W[\varsigma]+i\int_{\partial\Lambda}dt\,d\hat{\boldsymbol{x}}\,\varsigma\chi\right), \tag{48}$$

$$\frac{\delta W}{\delta\varsigma}=0,\quad R^2\frac{\partial\varsigma(R\hat{\boldsymbol{x}})}{\partial R}=i\chi, \tag{49}$$

where the contribution from the first Euler derivative of the action $\mathcal{E}^{(1)}[\varsigma]$ is precisely canceled with that from the surface term in Eq. (45).

Thus, in the mean-field approximation the dependence $Z[\chi]$ is controlled by the saddle point solution ς. As we have seen in the previous section, the Abelian gauge theory possesses only the trivial solution $\varsigma=0$.

In Ref. [9] we have studied constant solutions of the mean-field equations. Let us reproduce those results briefly here, but in addition carefully keeping a finite volume. For simplicity we also restrict ourselves to the gauge group SU(2). First of all, we introduce the notation for the free energy density $\mathcal{F}_R$:

$$W_R=\beta V_R\mathcal{F}_R,\quad \mathcal{F}_R=\gamma_RF_R,\quad \gamma_R=\frac{8\pi^2R\,\delta(\hat{0})}{\beta^2V_R}, \tag{50}$$

where $V_R=4\pi R^3/3$ is the domain volume and $\delta(\hat{0})=(1/4\pi)\Sigma_l(2l+1)$ is the ultravioletly divergent angular delta function with coinciding arguments. The function F_R is now expressed via the dimensionless variables

$$\sigma = \frac{2\pi s}{\beta g}, \quad \nu = i\left(\frac{2\pi u}{\beta g}\right)^2, \tag{51}$$

where to produce a real mean magnetic field ν has to be purely imaginary [see Eq. (44)]. After introducing the control parameter $a=(2\pi)^4/(2g^2\beta^4\gamma_R)$ and carrying out some derivations we obtain

$$F_R[u,s] = -au^4 + \mathcal{U}_R[u,s], \tag{52}$$

$$\mathcal{U}_R[u,s] = \mathcal{U}_R[s] + \mathcal{V}_R[u,s], \tag{53}$$

$$\mathcal{V}_R[u,s] = \frac{\beta}{2\pi R} \times \sum_{n=-\infty}^{\infty} \ln \frac{L_R((n+s)^2+u^2) L_R((n+s)^2-u^2)}{L_R^2((n+s)^2)}, \tag{54}$$

$$\mathcal{U}_R[s] = \frac{\beta}{\pi R} \sum_{m=0}^{\infty} \ln\left(1 - \frac{\cos 2\pi s}{\cosh[\pi(m+1/2)\beta/R]}\right), \tag{55}$$

where $L_R(x) = \cosh(2\pi R\sqrt{x}/\beta)$.

It can be seen that at finite R this free energy possesses only a trivial minimum at $s=u=0$. The situation changes after taking the thermodynamic limit $R\to\infty$. The resulting expression for the free energy density (see Ref. [9]) possesses only the trivial stable solution $u=s=0$ at high temperatures. However, at some critical temperature T_c the system undergoes a first-order phase transition, below which there appears a deeper nontrivial minimum at $u=s=1/2$ (see Ref. [9]).

Therefore, at high temperatures $T>T_c$ the dependence $Z[\chi]$ is determined by the solution of the linearized equation $\delta^2 W/\delta\varsigma^2 \bullet \sigma_1$ around $\varsigma=0$. This can only produce a dependence akin to the Abelian theory. Namely, it contains the delta function expressing the conservation of the global charge [Eq. (3) with $\rho=0$], and apart from that it is trivial in the sense that $Z[\chi_{lm}]\to 1$ in the thermodynamic limit $R\to\infty$. This situation, obviously, corresponds to the *deconfinement* phase as there is no restriction on the color fluxes at infinity.

However, below the critical temperature $T<T_c$ there is a nonzero constant solution $|\varsigma| = \pi/g\beta$. Since the system in invariant under the group of the *big gauge transformations* G_∞ parametrized by matrices $U(\hat{\boldsymbol{x}})$, the corresponding unit color vector $\hat{\varsigma}(\hat{\boldsymbol{x}})$ is arbitrary in every direction $\hat{\boldsymbol{x}}$. After integration over the orbits of the group SU(2) at each cone $\hat{\boldsymbol{x}}$ the dependence becomes

$$\tilde{Z}[\chi] = \prod_{\hat{\boldsymbol{x}}} \frac{\sin \Delta\beta\varsigma\chi(\hat{\boldsymbol{x}})}{\Delta\beta\varsigma\chi(\hat{\boldsymbol{x}})} \sim \exp\left(-\frac{\Delta\pi^2}{g^2}\int_{\partial V} d\hat{\boldsymbol{x}}\,\chi^2(\hat{\boldsymbol{x}})\right), \tag{56}$$

where we have introduced a discretization of the unit sphere with Δ being the infinitesimal cone area. It is well known [1] that in the continuous limit the bare coupling constant vanishes $g\to 0$, thereby making the effective partition function $Z[\chi]$ a very sharply peaked function around the zero argument due to its essentially nonperturbative dependence on g. This property corresponds to the *confinement* phase, in which color fluxes are equal to zero in every spatial direction at infinity.

So we can conclude that the dependence of the effective free energy on χ is the following:

$$Z[\chi] = \begin{cases} \prod_{\hat{\boldsymbol{x}}} \delta(\chi(\hat{\boldsymbol{x}})), & T<T_c, \\ 1, & T>T_c. \end{cases} \tag{57}$$

Therefore, the Gibbs average of an observable A is given by

$$\langle A\rangle = \begin{cases} \langle A\rangle_0, & T<T_c, \\ \int d\chi(\hat{\boldsymbol{x}})\langle A\rangle_\chi, & T>T_c, \end{cases} \tag{58}$$

where we have introduced the averages over inequivalent representations:

$$\langle A\rangle_\chi = \frac{1}{Z[\chi]}\int \mathcal{D}\sigma\,\mathcal{D}\nu \exp\left(-W[\sigma,\nu] + i\int_{\partial\Lambda} dt\, d\hat{\boldsymbol{x}}\,\sigma\chi\right) A[\sigma,\nu]. \tag{59}$$

It is straightforward to see that any Gibbs average at low temperatures contains the singlet projector of the group G_∞:

$$\langle A\rangle_0 = \lim_{R\to\infty}\frac{1}{Z_R[0]}\,\mathrm{Tr}[e^{-\beta H_R}\delta(Q_R)A] = \lim_{R\to\infty}\frac{1}{Z_R[0]}\,\mathrm{Tr}(e^{-\beta H_R}P_s A), \tag{60}$$

where $Q_R = \int_{V_R} d\boldsymbol{x}\,\nabla E$ is the operator of the color charge in volume V and P_s is the singlet projector of the big gauge transformations. The presence of this projector in the Gibbs averages has been demonstrated to lead to the area law for the Wilson loop [10,9], which is considered to be a standard confinement criterion.

V. CONCLUSION

The Gauss law in gauge field theory may be resolved explicitly in a physical gauge. This produces effectively nonlocal interactions generating a boundary nontriviality of the theory.

We have studied the dependence of the effective partition function on the Dirichlet boundary condition $R^2E_\parallel(R\hat{\boldsymbol{x}}) = \chi(\hat{\boldsymbol{x}})$ imposed on the residual component of the electric field for electrodynamics with an external charge and SU(2) gluodynamics. In the Abelian case this dependence always contains a delta function expressing the conservation of the total charge, and it is nontrivial only for charge distributions slowly decreasing at spatial infinity.

Non-Abelian self-interactions lead to a more unusual effect. Here the restriction of possible boundary values of the longitudinal electric field at low temperatures provides the confinement mechanism proposed by us in Ref. [9]. Indeed, this quantity is proportional to the flux of the electric field through an infinitesimal cone in the direction $\hat{\boldsymbol{x}}$ at infinity. Therefore, since the color flux vanishes for any direction, no color could escape to infinity and be experimentally observed.

Finally, let us discuss the status of the mean-field approximation involved in the last part of our derivations. In Refs. [9,10] we have found that the effective action is proportional to $W_R \sim (V_R/g^2\beta)\Xi_0 F_R$, where Ξ_0 is the string tension coefficient for the SU(2) group. Thus, there is a natural large parameter, which is infrared and ultraviolet divergent, which makes use of the saddle point approximation mathematically justifiable. Quite often in statistical mechanics a mean-field treatment in terms of proper collective variables yields a correct qualitative picture of phase transitions, though not necessarily quantitatively accurate.

To test this in Refs. [9,10] we have managed to calculate the ratio $\sqrt{\Xi_0}/T_c$, where T_c is the confinement transition temperature, and this ratio was found to be in fair agreement with data from lattice simulations. Despite bare cutoffs canceling out from some observables such as this one, their presence in others is an unavoidable limitation of any mean-field scheme. An account of higher order fluctuations should lead to an effective renormalization of all parameters of the theory, but it presents a considerable computational challenge due to the nonlocality and complicated realization of Poincaré invariance in the Fock-Schwinger gauge.

However, in our view the current approach presents a sufficiently simple and elegant theoretical scheme, which provides interesting insights into the mystery of confinement consistent with conventional wisdom. We believe that the main physical picture of the confinement-deconfinement transition discussed here should remain valid beyond the mean-field approximation. This is something that could be probed by lattice simulations if good procedures for studying mean color fluxes at infinity and other relevant observables can be implemented.

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APPENDIX: GENERALIZED FOCK-SCHWINGER GAUGE

Let V_R be a regular domain in $\boldsymbol{R}^3$ topologically equivalent to a ball with a smooth boundary ∂V_R. One can choose the curvilinear coordinate system $\boldsymbol{X}$ in V_R such that on the boundary ∂V_R the first coordinate is constant and equal to some parameter R, which would play the role of an infrared regulator, i.e., $\partial V_R = \{\boldsymbol{X}: X_1(\boldsymbol{x}) = R = \text{const}\}$. The field of vectors normal to the boundary for all possible values of R forms a differentiable vector field in $\boldsymbol{R}^3$. We shall denote the ordinary Cartesian coordinates as $\boldsymbol{x}$ to distinguish them from $\boldsymbol{X}$. The local orthonormal frame then can be written as

$$e_i^{(k)} = \frac{1}{h_k}\frac{\partial x_i}{\partial X_k}, \quad h_k = \left[\sum_{i=1}^{3}\left(\frac{\partial x_i}{\partial X_k}\right)^2\right]^{1/2}, \quad h \equiv \prod_{i=1}^{3} h_i, \tag{A1}$$

where h_k are called the Lamé coefficients. Also, to distinguish the components of vectors in the curvilinear frame we shall use the notation with indices in parentheses:

$$A_{(k)} = e^i_{(k)} A_i, \quad \partial_{(k)} \equiv \frac{1}{h_k}\frac{\partial}{\partial X_k}. \tag{A2}$$

Then $\boldsymbol{e}_{(1)}$ defines the field of normal vectors we have just introduced. It is natural to perform the $(2+1)$ decomposition onto the longitudinal and transversal (denoted by the Greek characters) components: $i \to (1,\alpha)$, $\alpha = 2,3$, where the corresponding 2D radius vector will be denoted as $\check{\boldsymbol{X}} = (X_2, X_3)$.

Gauge theory in a finite domain V_R acquires especially elegant formulation in a gauge which is consistent with the shape of the boundary. Namely, we shall require that the normal component of the gauge field vanish at every point:

$$\boldsymbol{e}_{(1)}(\boldsymbol{x})\boldsymbol{A}(t,\boldsymbol{x}) = 0, \quad \boldsymbol{A} = \boldsymbol{A}_\perp, \quad \boldsymbol{A}_\perp = P\boldsymbol{A},$$
$$P = \boldsymbol{1} - \boldsymbol{e}_{(1)} \otimes \boldsymbol{e}_{(1)}. \tag{A3}$$

This gauge condition is natural to name the *generalized Fock-Schwinger gauge*. Its most significant property is that the Gauss law $\nabla_i E_i = 0$ can be resolved explicitly,

$$E_{(1)} = -\frac{h_1}{h}\int_{X_1^{(0)}}^{X_1} dX_1' (h\Phi_\perp)(X_1', \check{\boldsymbol{X}}), \tag{A4}$$

expressing the longitudinal component of the electric field through the transversal components of the gauge and strength fields:

$$\Phi_\perp \equiv \nabla_i E_{\perp i} = \frac{h_\alpha}{h}\nabla_{(\alpha)}\left(\frac{h}{h_\alpha}E_{(\alpha)}\right). \tag{A5}$$

Above the lower integration limit $X_1^{(0)}$ is equal to some constant, which should be chosen in the reference point of our coordinate system. Analogously, one can solve the identity

$$\nabla_i G_i = 0, \quad G_i \equiv \nabla_j F_{ij}, \tag{A6}$$

expressing $G_{(1)}$ as

$$G_{(1)} = -\frac{h_1}{h}\int_{X_1^{(0)}}^{X_1} dX_1' (h \ \nabla_i G_{\perp i})(X_1', \check{\boldsymbol{X}}). \tag{A7}$$

Further, one of the components of the Bianchi identity is

$$\nabla_i B_i = 0, \quad B_k = \frac{1}{2}\epsilon_{ijk} F_{ij}, \tag{A8}$$

which allows us to express the longitudinal magnetic field as

$$B_{(1)} = -\frac{h_1}{h}\int_{X_1^{(0)}}^{X_1} dX_1' (h\nabla_i B_{\perp i})(X_1', \check{\boldsymbol{X}}). \tag{A9}$$

Other components of the Bianchi identity $\epsilon_{ijk} e^i_{(\alpha)} \nabla_j E_k = 0$ give

$$\frac{1}{h_\alpha}\frac{\partial}{\partial X_1}(h_\alpha E_{(\alpha)}) = \nabla_{(\alpha)}(h_1 E_{(1)}). \tag{A10}$$

A nice property of the Fock-Schwinger gauge is that the gauge strength and potentials are related to each other in a simple way. Indeed, from the definition of the gauge strength tensor, applying the gauge condition we have

$$\frac{1}{h_\alpha}\frac{\partial}{\partial X_1}(h_\alpha A_{(\alpha)}) = h_1 F_{(\alpha)(1)}. \tag{A11}$$

However, the integral form of this equation,

$$A_{(\alpha)} = \frac{1}{h_\alpha}\int_{\tilde{X}_1^{(0)}}^{X_1} dX_1' (h_1 h_\alpha F_{(\alpha)(1)})(X_1', \check{\boldsymbol{X}}), \tag{A12}$$

breaks the residual gauge transformations symmetry subgroup, whereas choosing the lower integration limits in all previous integral relations did not violate such a symmetry. Fixing some kind of the boundary condition above, e.g., choosing $\tilde{X}_1^{(0)} = x_1^0$ in the reference point so that

$$\lim_{X_1 \to x_1^0} (h_\alpha A_{(\alpha)})(\boldsymbol{X}) = 0, \tag{A13}$$

can be shown to be sufficient for determining a unique gauge field satisfying the gauge condition. Indeed, let $\tilde{\boldsymbol{A}}$ be any gauge field. The transformation to the Fock-Schwinger gauge is accomplished by a gauge transformation $U(\boldsymbol{X})$:

$$\boldsymbol{A}(\boldsymbol{X}) = U^{-1}[\tilde{\boldsymbol{A}}(\boldsymbol{X}) - g^{-1}\boldsymbol{\partial}]U(\boldsymbol{X}), \quad \boldsymbol{E}(\boldsymbol{X}) = U^{-1}\tilde{\boldsymbol{E}}(\boldsymbol{X})U(\boldsymbol{X}), \tag{A14}$$

which can be found from the equation

$$\frac{1}{h_1}\frac{\partial}{\partial X_1} U(\boldsymbol{X}) = g(\boldsymbol{e}_{(1)}\tilde{\boldsymbol{A}})(\boldsymbol{X})U(\boldsymbol{X}). \tag{A15}$$

For the moment we can choose the initial condition simply as $U(X_1 = 0, \check{\boldsymbol{X}}) = 1$. The solution of Eq. (A15) is given by the Dyson P-exponent:

$$U(\boldsymbol{X}) = P\exp\left(\int_0^1 d\alpha R(\alpha, \boldsymbol{X})\right),$$
$$R(\alpha, \boldsymbol{X}) = g X_1 (h_1 \boldsymbol{e}_{(1)}\tilde{\boldsymbol{A}})(\alpha X_1, \check{\boldsymbol{X}}). \tag{A16}$$

This can be explicitly worked out for the gauge fields:

$$\boldsymbol{A}^b(\boldsymbol{X}) = \tilde{\boldsymbol{A}}^a(\mathbf{X}) P\exp\left(\int_0^1 d\alpha[-g t^{abc} R^c(\alpha, \boldsymbol{X})]\right.$$
$$-g^{-1}\int_0^1 d\beta \boldsymbol{\partial} R^a(\beta, \boldsymbol{X})$$
$$\left.\times P\exp\left(\int_0^\beta d\gamma[-g t^{abc} R^c(\gamma, \boldsymbol{X})]\right)\right). \tag{A17}$$

By applying an additional residual gauge transformation we can always satisfy the boundary condition (A13), and uniquely. Really, suppose there exist two distinct gauge fields $\boldsymbol{A}'$ and $\boldsymbol{A}''$ satisfying Eqs. (A3),(A13) and such that $\boldsymbol{A}'' \neq U^{-1}\boldsymbol{A}' U, \quad \forall U = \text{const}$. Then there should exist a gauge transformation between the two:

$$\boldsymbol{A}''(\boldsymbol{X}) = U^{-1}(\boldsymbol{X})[\boldsymbol{A}'(\mathbf{X}) - g^{-1}\boldsymbol{\partial}]U(\boldsymbol{X}). \tag{A18}$$

Multiplication of this by $\boldsymbol{e}_{(1)}$ gives $(1/h_1)[\partial U(\boldsymbol{X})/\partial X_1] = 0$, i.e., $U = U(\check{\boldsymbol{X}})$. Such transformations form the subgroup G_{res} of the residual gauge symmetries in the Fock-Schwinger gauge. In fact, the boundary condition (A13) does not permit such a transformation since in the limit $\boldsymbol{X} \to \boldsymbol{x}^0$ in Eq. (A18) the gauge fields would have a singularity which is not compatible with such a boundary condition. This proves the uniqueness of the gauge field if condition (A13) is imposed.

As a simple example let us consider the *elliptic coordinates*: $1 \leqslant X_1 < \infty, \quad -1 \leqslant X_2 \leqslant 1, \quad 0 \leqslant X_3 \leqslant 2\pi$, where $X_3 = \phi$ is the polar angle and $X_1 = (r_1 + r_2)/2a, \quad X_2 = (r_1 - r_2)/2a$. Such a coordinate system is defined by two points located at distances $\pm a$ from the reference point along the z axis with $\boldsymbol{r}_1, \quad \boldsymbol{r}_2$ being the radius vectors from these points. The reference point is here $x_1^0 = 1, \quad x_2^0 = 0$ and the Lamé coefficients are given by

$$h_1^2 = a^2\frac{X_1^2 - X_2^2}{X_1^2 - 1}, \quad h_2^2 = a^2\frac{X_1^2 - X_2^2}{1 - X_2^2},$$
$$h_3^2 = a^2(X_1^2 - 1)(1 - X_2^2). \tag{A19}$$

In the limit $a = 0$ the ellipsoid becomes a sphere and the gauge turns into the standard Fock-Schwinger gauge. For the *spherical coordinates*: $X_1 = r, \quad X_2 = \phi, \quad X_3 = \theta$ the Lamé coefficients become very simple:

$$h_1 = 1, \quad h_2 = X_1 \sin X_3, \quad h_3 = X_1. \tag{A20}$$

A technically attractive property of this particular gauge is that $\boldsymbol{e}_{(i)}$ do not depend on X_1 and the vector normal to the boundary is just equal to the unit radius vector, $\boldsymbol{e}_{(1)} = \hat{\boldsymbol{x}}$.

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