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CONFINEMENT PHASE TRANSITION IN GLUODYNAMICS

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Abstract

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Here we continue the study of the effective free energy of SU(2)-gluodynamics. The earlier used *Ansatz* is shown to be the only one which provides stability. The generalization for the SU(3) group has been obtained.

Аннотация

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В данной работе продолжается изучение эффективной свободной энергии SU(2)-глюодинамики. Показано, что использовавшийся ранее анзац является единственным стабильным решением. Получено обобщение на группу SU(3).

Introduction

This paper is supposed to go through next two steps in the description of QCD confinement phase transition [1, 2]. With the exception of results of Appendix B it contains nothing of paramount importance but mainly technical details. Previously in the general framework of accurate formulation of Yang–Mills theory in 3-d Fock–Schwinger gauge the free energy density $F[\vec{s}, \vec{\nu}]$ was evaluated as a function of longitudinal in x-space magnetic field $\vec{\nu}$ and some extra phase parameter $\vec{s}$. That was made in the classical approximation over these collective variables which corresponds to taking into account an amount of quantum contributions in terms of the gauge fields. To begin with we simply investigate the most general case while looking for stable minima of the free energy in SU(2)-gluodynamics. Namely we do not restrict ourselves by the *Ansatz*

$$\vec{\nu}_\perp = 0\,, \qquad \vec{\nu}_\perp = (\mathbf{1} - \hat{\vec{s}} \otimes \hat{\vec{s}})\, \vec{\nu} \tag{1}$$

any longer. Though it turns out that at arbitrary nonzero $\vec{\nu}_\perp$ the free energy possesses imaginary part, and therefore the minimum is unstable.

Provided reliable background for (1), let us regard a more realistic case of SU(3) group. In the appropriate section necessary generalization is carried out in a quite obvious way. In fact, the whole picture is just the same as for SU(2). Only the phase transition temperature differs by the factor of $\sqrt{3/2}$.

Throughout we use the notations from [1, 2] and refer the reader there for details.

SU(2) group: general case

The general expression for the free energy density in terms of dimensionless parameters is given[1] by [1, 2]

$$
\begin{aligned}
F[\vec{s},\vec{\nu}] &= -a\,\vec{\nu}^{\,2} + \mathcal{U}[\vec{s},\vec{\nu}]\,, &(2)\\
\mathcal{U}[\vec{s},\vec{\nu}] &= \sum_n \int_0^\infty \frac{dv}{\pi} \log\det C[\vec{s},\vec{\nu}]\,, &(3)\\
C^{ab}[\vec{s},\vec{\nu}] &= \delta^{ab}\,(v^2+n^2) + i\,t^{abc}\,(2ns^c+\nu^c) + t^{acd}t^{bed}\,s^c s^e\,. &(4)
\end{aligned}
$$

Now we concentrate attention on our particular case. Using colour transformations it is easy to get the following parametrization

$$\vec{s} = (0,\,0,\,s)\,, \qquad\qquad \vec{\nu} = (\nu_\perp,\,0,\,\nu_\parallel)\,.$$

The determinant then is written as

$$\det C = X^3 + 2s^2X^2 + (s^4 - \vec{\nu}^2 - 4ns\,\nu_\parallel)\,X - \nu_\perp^2 s^2 \qquad (5)$$

with $X = v^2 + n^2$.

Let us denote by γ_a $(a = 1,2,3)$ the roots of the right–hand side of (5) over X. These roots will be investigated later on. By means of the formula

$$\int_0^\infty \frac{dv}{\pi} \log(1+\frac{a}{v^2}) = a^{1/2} \qquad (6)$$

we present function (4) as follows

$$
\begin{aligned}
\mathrm{Re}\,\mathcal{U}[s,\vec{\nu}] &= \sum_n [\sum_{a=1}^{3} (n^2 - \gamma_a[s,\vec{\nu},n])_+^{1/2} - 3\mid n \mid] - \frac{1}{2}\,, &(7)\\
\mathrm{Im}\,\mathcal{U}[s,\vec{\nu}] &= \sum_n \sum_{a=1}^{3} (\gamma_a[s,\vec{\nu},n] - n^2)_+^{1/2}\,. &(8)
\end{aligned}
$$

As far as $\mathcal{U}$ is periodic and even over s we consider it over half of the period $0 \le s \le 1/2$. First of all, the region where imaginary part exists must be excluded. Certainly the last thing is nonzero if

$$\gamma_a[s,\vec{\nu},0] > 0\,.$$

So we find out when the equation

$$x^2 + 2s^2\,x + s^4 - \vec{\nu}^2 = \frac{\nu_\perp^2 s^2}{x} \qquad (9)$$

[1] We have changed our notations slightly in order to accomodate them.

accepts positive or complex solutions. Everything is indicated very clearly on Fig. 1.

One can see that the lower branch of the hyperbolic curve $2''$ intersects parabola 1 in two points thus giving rise to real negative roots γ_2, γ_3. The following circumstance is of the most importance to emphasize. In any case when $\nu_\perp \neq 0$ the upper branch of hyperbola $2'$ intersects the parabola in positive point γ_1 therefore leading to instability. Since then the general case $\nu_\perp \neq 0$ presents no real interest.

Fig. 1 is so instructive that it even allows to discover the stable minimum of $F[\vec{s}, \vec{\nu}]$. The greater is $\mid \vec{\nu} \mid$ and the smaller is $\mid \gamma_a \mid$ conversely, the smaller this function will be. The negativity of roots ties us by the bound $\vec{\nu}^2 \leq s^4$, which will be saturated under $\mid \vec{\nu} \mid = \nu_{\|} = s^2 = \frac{1}{4}$. This is precisely what we know to be the case.

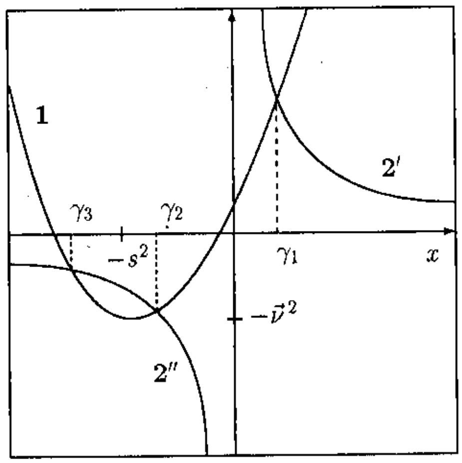

Fig. 1 On this picture the roots of the equation (9) are depicted. The points where the parabola **1** intersects the x-axis are equal to $-s^2 \pm \mid \vec{\nu} \mid$.

SU(3) group

Ansatz (1) being accepted the vectors $\vec{\nu}, \vec{s}$ can be transformed so that only their third and eighth components will be nonvanishing. If one introduces projections

$$s_\pm = \frac{1}{2} s_3 \pm \frac{\sqrt{3}}{2} s_8 \,, \qquad \nu_\pm = \frac{1}{2} \nu_3 \pm \frac{\sqrt{3}}{2} \nu_8 \tag{10}$$

and denotes

$$u^2 = \mid \nu_3 \mid, \qquad u_\pm^2 = \mid \nu_\pm \mid, \qquad s = s_3 \tag{11}$$

function (4) is expressed through functions (7, 8) of SU(2) group via

$$\mathcal{U}[\vec{s}, \vec{\nu}] = \mathcal{U}[s, u] + \mathcal{U}[s_+, u_+] + \mathcal{U}[s_-, u-]\,. \tag{12}$$

Without loss of generality we can regard $s, \nu, s_\pm, \nu_\pm \geq 0$. Since not all of parameters are independent due to

$$u_+^2 + u_-^2 = u^2\,, \qquad s_+ + s_- = s \tag{13}$$

it will be better to parametrize them as follows

$$u_+ = u\, \cos\varphi\,, \quad u_- = u\, \sin\varphi\,, \qquad -\frac{\pi}{2} < \varphi < \frac{\pi}{2} \tag{14}$$

$$s_+ = \frac{s + \tilde{s}}{2}\,, \quad s_- = \frac{s - \tilde{s}}{2}\,, \qquad 0 \leq s, \tilde{s} \leq \frac{1}{2}\,. \tag{15}$$

We write down (3) in the form

$$F[\vec{s}, \vec{\nu}] = -a\,(1 + \frac{1}{3}\cos^2 2\varphi)\, u^4 + \mathcal{U}[s, u] \;+\; \mathcal{U}[\frac{s+\tilde{s}}{2}, u\, \cos\varphi] + \\ + \;\mathcal{U}[\frac{s-\tilde{s}}{2}, u\, \sin\varphi]\,. \tag{16}$$

As minimum of sum is greater or equal than sum of minima, the first is achieved if there exists a compatible solution of

$$\cos^2 2\varphi = 1 \quad s = u\,, \quad s_+ = u_+\,, \quad s_- = u_- \tag{17}$$

and obviously $\varphi = 0, \tilde{s} = 0$ fits all right. One gets

$$F_{SU(3)} = -\frac{4}{3} a\, u^4 + 2\,\mathcal{U}[s, u]\,, \tag{18}$$

then introduces $\frac{2}{3}a = a_{SU(2)}$ and hence the phase transition temperature is

$$T_{SU(3)} = \sqrt{3/2}\; T_{SU(2)}\,. \tag{19}$$

Conclusion

This conclusion is devoted to the comparison of our results with lattice simulations. Recently the Wilson loop at confined phase was evaluated by one of the authors (E. T.) [3] for fundamental representation of SU(N) group:

$$\mathcal{W}_{SU(N)} = N \, \exp\Big\{-2\,(1-\frac{1}{N})\,\chi\, S\Big\}. \qquad (20)$$

Analogical values were numerically computed in pure gauge theory for SU(2) and SU(3) groups long ago [4-8]. In the first case the second order phase transition was observed. The energy density undergoes to fall an order of magnitude near $T \simeq 200 \; MeV$ and the specific heat per unit volume c_V has a peak at $T_c \simeq 215 \; MeV$ [6]. For SU(3) group there was a lot of debates about the order of the phase transition and finally it was argued that it is the first. So the energy density $\varepsilon(\beta)$ should have discontinuity at T_c. But really a sharp jump cannot be seen on computer.

In our approach we pointed out the discontinuity of the free energy density, i. e. the zero order phase transition in a way, regardless the group dimension. We prefer still to call it "the first order phase transition" as far as the equilibrium expectation value of magnetic field ν changes by jumping.

The following points are crucial:

i) the system is infinite and it is described by ultravioletely singular long–range interacting field theory;

ii) quantum bifurcation occurs due to dynamics at infinity [9].

The discontinuity of $\log Z$ therefore is not so surprising. None of these arguments is true in the lattice theory. Nevertheless it is a common belief that this theory can give correct physical values anyway. Indeed the processes in such regularized system proceed much more smoothly.

Finally, we write down one more result apart from those which are summarized in Tab. 1 (see Appendix A)

$$\xi_{SU(3)} \equiv \frac{T_c}{\sqrt{\chi}} = \frac{3}{\sqrt{8}\,\pi}\,\sqrt{a_c} \simeq 0.55\,, \qquad \text{on lattice: } \xi_{SU(3)} = 0.58 \pm 0.04\,. \quad (21)$$

Though the phase transition temperature is not the best thing to compare with in virtue of the above remarks, but not having yet anything else we qualify the agreement between two approaches as quite fair.

relations with our theoretical predictions.

relat.	lattice	theoretical
$\frac{T_c^{SU(3)}}{T_c^{SU(2)}}$	1.16 ± 0.07	$\sqrt{3/2} \simeq 1.22$
$\frac{\xi^{SU(3)}}{\xi^{SU(2)}}$	1.04 ± 0.18	$3/(2\sqrt{2}) \simeq 1.06$

We are grateful to E. E. Boos and V. I. Ilyin for useful discussions.

Appendix A

Precise description of the phase transition

In our first preprint [2] the confinement phase transition was described just draftly. We were satisfied at that stage with the weak assertion that at temperature tending to zero the singlet projector arises providing confinement. The explicit picture, as it turned out after consequent considerations, looks as follows.

The cuts in the complex plane of longitudinal magnetic field ν should be displaced as they are drawn on Fig. 2^a, 2^b rather than on Fig. 5 of [2].

Next, one can easily obtain the analytic formula for the critical value of the "a" parameter. Really, equating the right-hand side derivative of $F[s,u]$ over u nearby $\frac{1}{2}$ up to zero we'll arrive to the expression

$$a_c = 2\sqrt{2} + 4\sum_{k=1}^{\infty}\left[((2k+1)^2+1)^{-1/2} - ((2k+1)^2-1)^{-1/2}\right]. \qquad (A\,1)$$

It could be rewritten after expanding the inverse square roots in Teylor series

$$a_c = 2\sqrt{2} - 8\sum_{k=0}^{\infty} \frac{(4k+1)!!}{(2k+1)!\,2^{k+1}} \left[(1 - \frac{1}{2^{4k+3}})\,\zeta(4k+3) - 1\right]. \qquad (A\ 2)$$

The latter ζ-functions series diverges very quickly to the value $a_c \approx 2.61882\ldots$

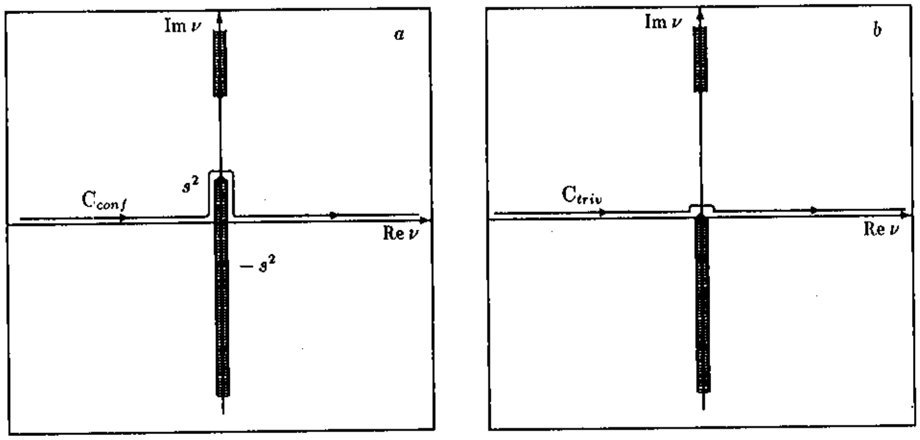

Fig. 2 a) The saddle point contour in the ν complex plane at $s = 0.5$
b) The saddle point contour in the ν complex plane at $s = 0$.

Finally, we would like to propose the most adequate construction of the equilibrium state at finite temperatures. For the sake of convenience we formulate this result in a separate Appendix.

Appendix B

Structure of the equilibrium state

Let us remind the reader that the evolution invariant equilibrium state includes the integration over the **variables at infinity**[2]

$$Z = \int_{G_\infty} d\varsigma_\infty\, e^{-\beta\, h(\varsigma_\infty(\hat{\mathbf{x}}),\sigma_0)}\, \mathrm{Tr}_{\mathcal{H}_0}(e^{-\beta H}\, e^{i\,\Phi(\varsigma_\infty)}), \qquad (B\ 1)$$

[2]Here we use the dimensionless variables $\vec{\varsigma}_\infty(\hat{\mathbf{x}}) \equiv \beta\,\vec{\sigma}_\infty(\hat{\mathbf{x}})$.

Where h is the Hamiltonian of dynamics at infinity constructed by means of the Kirillov canonical structure [2]

$$h(\varsigma_\infty(\hat{\mathbf{x}}),\sigma_0) = -i\,\frac{g}{2\pi}\int d\hat{\mathbf{x}}\;\vec{\sigma_0}\,\vec{\varsigma}_\infty(\hat{\mathbf{x}})\,. \qquad (B\ 2)$$

The integration has to be performed over the Haar measure of the group of *gauge transformations at infinity.* In the SU(2) case it takes a form

$$\int_{G_\infty} d\varsigma_\infty \equiv \int_0^\pi \prod_{\hat{\mathbf{x}}} \sin^2\varsigma_\infty(\hat{\mathbf{x}})\,d\varsigma_\infty(\hat{\mathbf{x}}) \;\Big/ \prod_{\hat{\mathbf{x}}} d\hat{\varsigma}_\infty(\hat{\mathbf{x}})\,.$$

The trace above can be presented in terms of the path integral

$$Z[\beta^{-1}\varsigma_\infty] = \int_{\leftrightarrow} \mathcal{D}\sigma\,\mathcal{D}\nu\; e^{-W[\sigma+\beta^{-1}\varsigma_\infty,\nu]}\,. \qquad (B\ 3)$$

The effective action $W[\sigma,\nu]$ (see (55) in [2]) is invariant under the following time–depending gauge transformations:

$$\nu \to \nu' = U^{-1}(t)\,\nu\,U(t)\,, \qquad \sigma \to \sigma' = U^{-1}(t)\,\sigma\,U(t)\,, \qquad (B\ 4)$$

$$\varsigma_\infty \to \varsigma'_\infty = U^{-1}(t)\,(\varsigma_\infty + g^{-1}\,\partial_t\,)\,U(t)\,. \qquad (B\ 5)$$

Surely, if one wants to preserve the periodic boundary conditions the relation for $U(t)$ matrix

$$U(\beta) = U(0) \qquad (B\ 6)$$

should be kept. The trick is to choose this matrix in the form emulating the dynamics at infinity

$$U(t) = e^{g\,\sigma_0\,t\,\hat{\eta}^a T^a}\,, \qquad g\,\beta\,\sigma_0 = 2\pi\,n \qquad (B\ 7)$$

with rotation frequency σ_0 in agreement with (B 6). Due to invariance of the action the integrals measure being invariant too upon uniform transformations of the kind, allows one to get the *triviality condition* determining different vacuum sectors

$$Z[\beta^{-1}\,\vec{\varsigma}_\infty + \sigma_0\hat{\vec{\eta}}] = Z[\beta^{-1}\,\vec{\varsigma}_\infty]\,,$$

where we've taken into account that $U^{-1}(t)\,\vec{\varsigma}_\infty\,U(t)$ coincides with the true dynamics generated by $h(\vec{\varsigma}_\infty, \sigma_0\hat{\vec{\eta}})$. No doubts, the periodicity points of $Z[\varsigma_\infty]$' s are just the same as it was required earlier.

As long as the contributions from sectors with different n contrast one from another no more than by a phase it'll be quite natural to sum over them

$$Z = \int_{G_\infty} d\varsigma_\infty \sum_{n=0}^{\infty} e^{i\,n\,\int d\dot{\mathbf{x}}\,\vec{\varsigma}_\infty(\dot{\mathbf{x}})\hat{\vec{\eta}}}\, \mathrm{Tr}_{\mathcal{H}_0}\big(\, e^{-\beta\, H}\, e^{i\,\Phi(\varsigma_\infty)}\big)\,. \qquad (B\ 8)$$

Using the fact

$$2\ \mathrm{Re} \sum_{n=0}^{\infty} e^{i\,n\,\theta} = 1 + 2\,\pi \sum_{n=-\infty}^{\infty} \delta(2\pi\, n - \theta) \qquad (B\ 9)$$

we can observe that all the remembrance of the variables at infinity dynamics disappears from such a θ*-vacuum* partition function. The singular contribution in $(B\ 9)$ vanishes due to the specific construction of the Haar measure.

Hence the singletness is a rigour property at any $T < T_c$

$$Z_s = \mathrm{Tr}(P_s\, e^{-\beta\, H})\,, \qquad P_s = \int_{G_\infty} d\varsigma_\infty\ e^{i\,\Phi(\varsigma)}\,,$$

where the last integration is performed over the group. So far *the confinement phase transition is an sharp change and the equilibrium state is always gauge–invariant in the whole temperature range.*

This conclusion seems to be more attractive than the previous scenario of *delayed confinement* from both either experimental and esthetic points of view. It demonstrates in the explicit way that our particular Fock–Schwinger gauge plays no distinguished role amidst others not only at zero temperature, but rather at any. We might always transform it in a way we'd like to and we should not care about symmetry conservation as well.

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