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CONFINEMENT PHASE TRANSITION MECHANISM OF SU(2)-GLUODYNAMICS

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Abstract

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We give the formulation of QCD in 3-dimensional Fock–Schwinger gauge in operatorial as well as in functional languages. The appearance of variables at infinity is retraced in the equations of motion. The partition function of gluodynamics is represented in terms of a functional integral over collective (chromo) electric and magnetic fields. The first order phase transition confinement–deconfinement for SU(2) group is derived from first principles. The general mechanism underlying it is clarified.

Аннотация

Н. А. Свешников, Э. Г. Тимошенко. МЕХАНИЗМ ФАЗОВОГО ПЕРЕХОДА КОНФАЙНМЕНТ–ДЕКОНФАЙНМЕНТ SU(2)-ГЛЮОДИНАМИКИ: Препринт ИФВЭ 91-140. — Протвино, 1991. — 30 с., библиогр.: 24.

Построены операторная и функциональная формулировки КХД в калибровке Фока-Швингера. Прослежено возникновение переменных на бесконечности в уравнениях движения. Статистическая сумма глюодинамики представлена в виде функционального интеграла по коллективным (хромо) электрическому и магнитному полям. Из первых принципов выведен фазовый переход конфайнмент-деконфайнмент для группы SU(2), объясняется его механизм.

1. INTRODUCTION

The confinement property seems to be the most complicated puzzle of Yang–Mills fields for years. The perturbation theory in covariant (α-type) gauges fails to describe this phenomenon up to now, which may be not perturbative at all. In the present paper we suggest a nonperturbative approach throwing some light upon the mechanism of confinement.

In order to make the Hilbert space positive definite we choose a gauge dealing with physical degrees of freedom only [1]. In nonabelian case a serious problem arises: how to guarantee complete gauge fixing without Gribov copies [2] having Hamiltonian sufficiently simple for work which is acceptable for some kind of nonperturbative treatment. There also arises a vague point about boundary conditions for fields at spatial boundary specifying the definition of gauge [3] and near time–infinity in conformity with certain assumptions what asymptotic states are [4].

Fortunately, the favoured gauge, which fits all these requirements, is well known. This is the coordinate or the Fock–Schwinger gauge [5, 6, 7]:

$$\mathbf{x}\mathbf{A}^a(t,\mathbf{x}) = 0\,. \qquad (1)$$

The constraint, i. e. the Gauss law is exactly solvable in this gauge without using series expansion and then after being substituted into the Hamiltonian provides its nice form as a fourth–order polynomial of physical fields. It is worthwhile to stress here, that we mean namely three–dimensional version of the gauge, rather than a covariant one having many applications in QCD [8, 9]. Though there are many features in common for both of them, such as still only the noncovariant version is a physical gauge in our sense.

This fact has an important sequel called "dynamics diagonalization" Postponing its precise formulation for a little while, we only say, that the path integral can be reexpressed in terms of two essentially nonabelian strength components in a explicit way, rather than all six strengths as in the standard covariant field–strength approach [11, 12]. These two components are collective nonlocal objects.

Now let us briefly discuss the boundary conditions problem mentioned earlier. The gauge (1) is manifestly invariant under 3-dimensional rotations. Thus it should impose the boundary conditions at zero point and near infinity. There exists a natural choice

$$\mathbf{A}^a(t,0) = 0 \qquad (2)$$

which enables complete gauge arbitrariness fixing and provides self–adjointness of an appropriate operator in the path integral.

The crucial trial for our approach is the vanishing of mixed boundary terms containing both gauge and collective variables. It would seem at first glance that they can't be zero. In fact, these terms are vanishing due to condition (1), otherwise the theory will be inconsistent. The last statement as well as the Gauss law solution, dynamics diagonalization and completeness of gauge fixing are the bulk arguments in fávour of the Fock–Schwinger gauge amongst many others.

The gauge, in turn, has a list of disadvantages: noncovariantness, nonlocality and even noninvariance under 3-d translations. Although these are only defects of appearance and at observables level that's all right with these symmetries [1], however the renormalization in the coordinate gauge is very tedious owing to the complicated structure of bare gluon propagator in it (see Appendix B).

A thoughtful deliberation of the S-matrix theory would indicate one more bewilderment. The asymptotic states of QCD are forbidden by confinement to be simply free waves of gluons and quarks. Hence we arrive at the following dilemma. It is impossible to evaluate the S-matrix without knowing them on the one hand, and on the other hand the states can be obtained not until the full solution of dynamics. We regard just a trite detour rather than a way–out of the problem.

Consider the thermal QCD where all fields in the path integral must be periodic (antiperiodic for fermions) over time within an interval from zero possesses thereto an advantage of noticing a picture of phase transition if it really takes place.

The main result of the presented research concerns of detailed realization what confinement is and how it occurs. First of all, let's give the definition. We say that *the Gibbs state at inverse temperature* β *is* confined, *if the correlation functions are restricted by singlet sector, i. e. they include the orthogonal projector onto the local singlet subspace* $\mathcal{H}_s$ *of the universal representation Hilbert space* $\mathcal{H}$.

So we shall try to show that the projector P_s arises within $\langle F \cdots G \rangle_\beta$ if $T < T_c$ and it becomes colourless then. This phase transition turns out to be of the first order, when the free energy $\mathcal{F}$ jumps through discontinuity into energetically more favoured state. As usual it happens just as the order parameters acquire nonzero expectations values. There are two of them in our case: a (chromo) magnetic field ν [13] and an exclusively quantum phase parameter s ($0 \leq s \leq 2\pi$). Really, the variable like this phase has a precedent in the Aharonov–Bohm effect [14]. The quantum phase over there exhibits nontrivial space topology under an abelian gauge group, otherwise the topological structure of SU(N) group itself entails the origin of a new number in spite of the space connectness.

The last quantity is related with variables at infinity which were first introduced in the algebraic quantum field theory [15, 16]. Although a special care has been rendered to grant the theory with a good Hilbert space, nevertheless one nuisance springs from reasons of great physical importance. The long range interactions and nonlocality evoke the appearance of variables at infinity in the equations of motion.

Such variables possess dynamics compelling one to expand the mathematical formalism of quantum theory whose description ought to be brought about by the von Neumann algebra with a nontrivial center. The ideas, mathematical apparatus as well as some physical consequences of the long–range dynamics were explored in the prominent works by Morchio and Strocchi [17, 18, 19]. Meanwhile, that nice toy–models studied at the finest level of mathematical stringency were far from the realistic quantum field theory in 4-dimensions with all its divergences. We have undertaken an effort to reconcile the generality of new mathematics with the pragmatic functional method of an effective action.

The dot outline of confinement cause consists in the following. Through variables at infinity the dynamical equations depend upon the representation quantum effects generate bubble formation within nonzero magnetic field and "s" inside it while the gluonic plasma has quenched below the critical temperature accompanied with the change of usual thermal Gibbs mean values towards the insertion of an extra averaging factor over variables at infinity like

$$P_s = \int d\,\sigma_\infty(\hat{\mathbf{x}})\;\exp[\,i\,\Phi_\perp(\sigma_\infty)] \tag{3}$$

into them on account of the representation variance. Here $\Phi_\perp$ is the residual gauge transformations operator commuting with the Hamiltonian of the system and P_s is just the local singlet projector.

We derive afterwards the effective free energy of pure SU(2) gluodynamics for the sake of simplicity as a function of collective variables. The generalization for the case of full QCD is only a matter of technique and it will be carried out probably in the next our publications.

The paper is organized as follows. In section 2 we introduce our notations and discuss the gauge fixing procedure especially on a classical level. The third section is devoted to Hamiltonian formulation and to the problem of an infrared cut–off removal, which entails that variables at infinity come into play. In section 4 we derive the partition function expression integrating exactly over gauge fields. The boundary terms are controlled carefully all the way. The following section is about the renormalization in the effective action performed by the integration with respect to "fast" variables. Section 6 is of the most importance. We write down the free energy of SU(2) Yang–Mills theory in the saddle point method. In the last division we shall investigate confinement–deconfinement phase transition. The free energy has a minimum corresponding to the confined phase if the temperature is less than

$$T_c \simeq \; 2\alpha_s^{1/2}\,\Lambda\,. \tag{4}$$

Our approach is developed on the ground of Yang–Mills field theory and we do not use model approximations of any kind.

2. FOCK–SCHWINGER GAUGE CONDITION

Let us establish some formal notations which will be convenient further. Throughout the paper we denote 3-dimensional normalized vectors by means

$$\hat{\mathbf{x}} = \mathbf{x}/x\,, \qquad x = |\,\mathbf{x}\,|\,, \tag{5}$$

their transversal and longitudinal components will be marked as subscript in the case of coordinate or x-space and as superscript when the momentum k-space implied. SU(2) adjoint representation vector space is called colour (c-space) with an arbitrary normalized vector $\hat{\eta}$. The orthogonal projection operators in all the spaces are

x-space

$$P_{ij} = \delta_{ij} - \hat{x}_i\hat{x}_j\,, \quad \mathbf{a}_\perp = P\mathbf{a}\,, \quad \mathbf{a}_\| = (1 - P)\mathbf{a}\,, \tag{6}$$

$$L_{ij} = \varepsilon_{ijk}\hat{x}_k\,, \quad L^2 = -P\,, \quad LP = PL = L\,, \tag{7}$$

$$\Pi_\pm = \frac{1}{2}(P \pm i\,L)\,, \quad \Pi_\pm = \Pi_\pm^2\,, \quad \Pi_+\Pi_- = 0\,, \quad \Pi_+ + \Pi_- = P\,, \tag{8}$$

$$\mathbf{a}_\pm = \Pi_\pm\mathbf{a}\,, \quad \partial_\pm = \Pi_\pm\partial\,, \quad \hat{\partial} = x\,P\partial\,. \tag{9}$$

k-space

$$Q_{ij} = \delta_{ij} - \partial_i\,\Delta^{-1}\partial_j\,, \quad \mathbf{a}^\perp = Q\mathbf{a}\,, \quad \mathbf{a}^\| = (1 - Q)\mathbf{a}\,. \tag{10}$$

c-space

$$\mathcal{P}^{ab} = \delta^{ab} - \hat{\eta}^a\hat{\eta}^b\,, \qquad \mathcal{L}^{ab} = \epsilon^{abc}\hat{\eta}^c\,. \tag{11}$$

The notations of functional space domains are: $X_\|$, $X_\perp$ in x-space and $K^\|$, $K^\perp$ in k-space. Besides S_R, K_R will be 3-dimensional sphere and ball of radius R, Ω — euclidean cylinder $\{|\,\mathbf{x}\,| < R,\ 0 < t < \beta\}$. We shall regard x-differential operators

$$\partial_x = \hat{\mathbf{x}}\partial = \partial\,/\,\partial\,x\,, \qquad \partial\hat{\mathbf{x}} = \partial_x + 2/x\,, \tag{12}$$

$$\Delta_x = x^{-1}\partial_x^2\,x\,, \qquad \Delta = \partial^2 = \Delta_x + x^{-2}\hat{\Delta} \tag{13}$$

with inverse

$$[\,\Delta^{-1}\,\mathbf{f}\,]\,(x) = -\int d\mathbf{x}'\,(\,4\pi\,|\,\mathbf{x} - \mathbf{x}'\,|\,)^{-1}\mathbf{f}(\mathbf{x}')\,, \tag{14}$$

$$[\,(\partial\hat{\mathbf{x}})^{-1}\mathbf{f}\,]\,(\mathbf{x}) = x^{-2}\int_0^x y^2dy\,\mathbf{f}(y\hat{\mathbf{x}})\,. \tag{15}$$

with the convention to use a hat as only angular components are concerned. Finally let us introduce these linear integration x-operators

$$[\mathrm{K}\,\mathbf{f}]\,(\mathbf{x}) = \mathbf{f}(\mathbf{x}) - \partial \int_0^x dy\,\hat{\mathbf{x}}\mathbf{f}(y\hat{\mathbf{x}})\,, \tag{16}$$

$$[\mathrm{G}\,\mathbf{f}]\,(\mathbf{x}) = \mathbf{f}(\mathbf{x}) - \hat{\mathbf{x}}\,x^{-2} \int_0^x y^2 dy\,[\,\partial\,\mathbf{f}\,]\,(y\hat{\mathbf{x}}) \tag{17}$$

obeying the algebra

$$\mathrm{KP} = \mathrm{P}\,, \quad \mathrm{PK} = \mathrm{K}\,, \quad \mathrm{QK} = \mathrm{Q}\,, \quad \mathrm{KQ} = \mathrm{K}\,, \tag{18}$$

$$\mathrm{GP} = \mathrm{G}\,, \quad \mathrm{PG} = \mathrm{P}\,, \quad \mathrm{QG} = \mathrm{G}\,, \quad \mathrm{GQ} = \mathrm{Q}\,. \tag{19}$$

We are going to investigate the gauge fixing properly. The Fock-Schwinger gauge condition is

$$\hat{\mathbf{x}}\mathbf{A}_{FS}(t, \mathbf{x}) = 0\,, \tag{20}$$

or in other words

$$\mathbf{A}_{FS} = \mathbf{A}_{\perp}\,,$$

where in (20) the vanishing of $\mathbf{A}_{FS}$ at zero is implied. The motive of such a boundary condition choice is given by

> Statement. *There exists a unique element of a gauge transformations group providing the transformation from an arbitrary chosen gauge field to such that equation (20) holds, whose all solutions are connected by rigid transformations only, i. e. condition (20) enables the fixation of one Lie–algebra element in any given gauge orbit.*

For the first part of this statement see Appendix A. Let us assume the existence of two different fields such that

$$\hat{\mathbf{x}}\mathbf{A}''_{FS} = \hat{\mathbf{x}}\mathbf{A}'_{FS} = 0 \quad \text{with} \quad \mathbf{A}''_{FS} \neq U^{-1}\mathbf{A}'_{FS}U \qquad \forall\, U = \text{const}\,.$$

Then there should exist the gauge transformation connecting them

$$\mathbf{A}''_{FS}(\mathbf{x}) = U^{-1}(\mathbf{x})\;(\mathbf{A}'_{FS}(\mathbf{x}) - g^{-1}\,\partial\,)\;U(\mathbf{x})\,. \tag{21}$$

Multiplication by $\hat{\mathbf{x}}$ gives $\partial\, U(\mathbf{x})\,/\,\partial\, x = 0$, i. e. $U = U(\hat{\mathbf{x}})$. After substitution $x = 0$ in (21) we arrive to the contradiction with the boundary condition, which proves that FS gauge field is ... fulfils under the above assumptions and Bianchi identities

$$A^i_{FS}(t, \mathbf{x}) = \int_0^1 \alpha\, d\alpha\, F^{ij}_{FS}(t, \alpha\mathbf{x})\, x_j \,. \tag{22}$$

The Gauss law constraint

$$\Phi^a = \nabla\mathbf{E}^a - gJ_0^a = 0\,, \qquad \nabla\mathbf{E}^a = \partial\mathbf{E}^a - gt^{abc}\mathbf{A}^b\mathbf{E}^c \tag{23}$$

can be solved in FS gauge exactly. This follows from an obvious fact $\mathbf{A}^b_\perp\mathbf{E}^c = \mathbf{A}^b_\perp\mathbf{E}^c_\perp$. The simple differential equation

$$(\partial\hat{\mathbf{x}})\, E^a_{\|} = -\Phi^a_\perp(\mathbf{x}) = -\nabla\mathbf{E}^a_\perp + gJ^a_0$$

has the solution

$$E^a_{\|}(\mathbf{x}) = -x^{-2}\int_0^x y^2\, dy\, \Phi^a_\perp(y\hat{\mathbf{x}})\,, \tag{24}$$

where the lowest integral limit is determined by demanding the absence of $1/\,x^2$-singularity at zero.

After substitution of (24) into QCD Hamiltonian it becomes quartic over $\mathbf{A}_\perp$, $\mathbf{E}_\perp$. But there is a price to pay off — nonlocality even in the squared part of Hamiltonian $H_{(2)}$. How to diagonalize it ?

One introduces a momentum variable $\mathbf{E}_D$ such that $\partial\mathbf{E}_D = gJ_0$ then the Gauss law preserves under the anzats

$$\mathbf{E}^a_{FS}(\mathbf{x}) = \mathbf{E}^a_D(\mathbf{x}) + \frac{\hat{\mathbf{x}}}{x^2}\int_0^x y^2\, dy\, gt^{abc}[\,\mathbf{A}^b_\perp\mathbf{E}^c_{FS}\,](y\hat{\mathbf{x}})\,,$$

which can be reversed using (24)

$$\mathbf{E}_D = \mathbf{E}_\perp - \hat{\mathbf{x}}\,(\partial\hat{\mathbf{x}})^{-1}(\Phi_\perp + [\mathbf{A}_\perp, \mathbf{E}_\perp])\,. \tag{25}$$

Decompose the diagonalizing variables on k-transversal components

$$\mathbf{A}_D = \mathbf{A}^\perp\,, \qquad \mathbf{E}_D = \mathbf{E}^\perp + \partial\,\varphi\,, \qquad \varphi = \Delta^{-1}\, gJ_0\,.$$

Theorem. *Let* $\mathbf{A}_\perp$, $\mathbf{E}^\perp$ *obey the boundary conditions*

$$\mathbf{A}_\perp(0) = \mathbf{0}\,, \qquad \lim_{x\to 0} x^2\, \mathbf{E}^\perp(\mathbf{x}) = 0\,,$$

then the connection formulae between two sets of variables can be briefly summarized in form (26, 27). The maps

$$\mathrm{K}: K^{\perp} \to X_{\perp}\,, \qquad \mathrm{Q}: X_{\perp} \to K^{\perp}\,,$$

$$\mathrm{P}: K^{\perp} \to X_{\perp}\,, \qquad \mathrm{G}: X_{\perp} \to K^{\perp}$$

are one–to–one, thus the Jacobian of this transformation is a nonzero normalization constant.

$$\mathbf{A}_{\perp} = \mathrm{K}\,\mathbf{A}^{\perp}\,, \qquad \mathbf{A}^{\perp} = \mathrm{Q}\,\mathbf{A}_{\perp}\,, \tag{26}$$

$$\mathbf{E}_{\perp} = \mathrm{P}\,(\mathbf{E}^{\perp} + \partial\,\varphi)\,, \qquad \mathbf{E}^{\perp} = \mathrm{G}\,(\mathbf{E}_{\perp} - \partial\,\varphi)\,. \tag{27}$$

The substitution of (26, 27) turns $H_{FS(2)}$ into exactly QED Coulomb gauge Hamiltonian in terms of physical transversal modes. This transformation is a canonical one, i. e. it preserves the Poisson bracket

$$[\,A^i_{\perp}(\mathbf{x}), E^j_{\perp}(\mathbf{y})\,] = i\,(\,\mathrm{KQP}\,)_{ij}(\mathbf{x},\mathbf{y}) = i\,\mathrm{P}_{ij}\,\delta(\mathbf{x}-\mathbf{y})\,. \tag{28}$$

Though these results can quite easily be obtained, still they have not been investigated, to the best of our knowledge, in the previous literature yet. Such odd circumstance becomes clear, if we remember that 3-d FS gauge is not suitable in abelian theory. Only its nonperturbative efficiency has stimulated our research on account of general topics.

3. HAMILTONIAN FORMALISM AND VARIABLES AT INFINITY

We merely explain here a mechanism of physical effect in the language reach enough to contain unusual possibilities and do not pretend on mathematical rigour.

In the algebraic QFT framework [16, 15] dynamics under retained cut–off is a 1-parameter automorphism group α^t_R of a quasilocal algebra $\mathcal{A}$ generated by cut–off Hamiltonian $H_R \in \mathcal{A}$. Let's consider a class of states $\mathcal{F}$ as in [18] and denote $w_{\mathcal{F}}$ the weak topology defined by $\mathcal{F}$ on double commutant $\mathcal{A}''$, besides $\mathcal{M}$ will be an extension of $\mathcal{A}$ in $w_{\mathcal{F}}$. There exist $w_{\mathcal{F}}\text{-}\lim_{R\to\infty} \alpha^t_R[A] = \alpha^t[A] \in \mathcal{M} \quad \forall\, A \in \mathcal{M}$ and α^t is an automorphism group of $\mathcal{M}$. Dynamical system is a triple $(\,\mathcal{M}\,, \mathcal{F}\,, \alpha^t\,)$. It is important to stress that in the presence of long–range interactions algebraic dynamics can be defined only on $\mathcal{M}$ with a nontrivial center $\mathcal{Z} = \mathcal{M} \cap (\,\cap_R \mathcal{A}'_R\,)$ generated by variables at infinity localization of dynamics on subalgebra $\mathcal{A}_l \subset \mathcal{M}$ accepting conditions like in [18] which include the existence of an automorphisms α^t_π of $\mathcal{A}_l$ such that $\forall\ \phi \in \pi \quad \phi(\alpha^t[A]) = \phi(\alpha^t_\pi[A]) \quad \forall\ A \in \mathcal{A}_l$. The symmetries of a system are naturally defined as an automorphism group β^ς of $\mathcal{M}$ commuting with the time evolution.

The $\perp$-symbol will be omitted since now if this can not lead to confusion. The evolution α^t_R for Yang–Mills system in FS gauge is generated by cut–off Hamiltonian

$$H_R = \frac{1}{2}\int d\mathbf{x}\, f_R(x)\,[\,(\mathbf{E}^a)^2 + (E_{\|})^2 + \frac{1}{2}(F^a_{ij})^2\,]\,, \tag{29}$$

$$E^a_{\|}(\mathbf{x}) = -x^{-2}\int_0^x y^2\,dy\,\Phi^a(y\hat{\mathbf{x}})\,, \quad \Phi^a = \nabla\mathbf{E}^a = \partial\mathbf{E}^a - gt^{abc}\mathbf{A}^b\mathbf{E}^c\,,$$

$$F^a_{ij} = \partial_j A^a_i - \partial_i A^a_j + gt^{abc}A^b_i A^c_j$$

with some smooth function

$$f_R(x) = \begin{cases} 1, & x \le R' \\ 0, & x \ge R''\,, \end{cases} \qquad \begin{array}{l} R' = (1-\varepsilon)R\,, \\ R'' = (1+\varepsilon)R\,. \end{array}$$

Canonical commutator is

$$[\,A(g_1)\;,\;E(g_2)\,] = i\,(g_1\;,\;g_2)\,,$$

$$(g_1\;,\;g_2) = \int d\mathbf{x}\,g^a_{1i}(\mathbf{x})\,P_{ij}(\hat{\mathbf{x}})\,g^a_{2j}(\mathbf{x}) \qquad \forall\ g_A \in \mathcal{G}(\mathbf{R}^3) \quad (A = 1,2)\,.$$

The equations of motion $\mathbf{A}(t) = \alpha^t_R[\mathbf{A}]$, $\mathbf{E}(t) = \alpha^t_R[\mathbf{E}]$ follow from (29) immediately:

$$\dot{A}^a_k = E^a_k - P_{kl}\nabla_l\tilde{\sigma}^a_R\,, \qquad \dot{E}^a_k = gt^{abc}E^b_k\tilde{\sigma}^c_R - P_{kl}\nabla_i F^a_{il}\,, \tag{30}$$

$$\tilde{\sigma}^a_R(\mathbf{x}) = \int_x^\infty dy\, f_R(y)\frac{1}{y^2}\int_0^y z^2\,dz\,\Phi^a(z\hat{\mathbf{x}})\,. \tag{31}$$

It is easy to check the commutational relation

$$[\,\Phi^a(\mathbf{x})\;,\;\Phi^b(\mathbf{x}')\,] = -i\,gt^{abc}\Phi^c(\mathbf{x})\,\delta(\mathbf{x}-\mathbf{x}')\,, \tag{32}$$

where the identity $\nabla_i\nabla_j F_{ij} = 0$ was used. From (32) one can conclude that

$$[\,H_R\;,\;\Phi(\varsigma)\,] = 0 \quad \forall\ \varsigma(\hat{\mathbf{x}})\,, \qquad \Phi(\varsigma) = \int d\mathbf{x}\,\Phi^a(\mathbf{x})\,\varsigma^a(\hat{\mathbf{x}}) \tag{33}$$

gauge transformations group G

$$- i\,[\,\Phi(\varsigma)\ ,\ \mathbf{A}^a(\mathbf{x})\,] = \nabla\varsigma^a(\hat{\mathbf{x}}) \qquad \forall\, \varsigma^a(\hat{\mathbf{x}}) \tag{34}$$

and in accordance with (33) this is a symmetry group β^ς commuting with the cut–off dynamics $\alpha^t_R\beta^\varsigma = \beta^\varsigma\alpha^t_R$. The choice of (2) condition prohibits such transformations because of their singular behaviour.

In our case $\mathcal{A}_l$ contains composite field σ besides $\mathbf{A},\mathbf{E}$

$$\tilde{\sigma}^a_R(\mathbf{x}) = f_R(\mathbf{x})\sigma^a(\mathbf{x}) - \sigma^a_R(\mathbf{x})\,, \quad \sigma^a_R = \int_x^\infty dy\, f_R{}'(y)\sigma^a(y\hat{\mathbf{x}})\,, \tag{35}$$

$$\sigma^a = -(\,\Delta_x^{-1}\,\Phi^a\,)(\mathbf{x}) = \int_x^\infty \frac{dy}{y^2}\int_0^y z^2\,dz\,\Phi^a(z\hat{\mathbf{x}})\,. \tag{36}$$

In the $R \to \infty$ limit σ^a_R includes delocolized variables. The algebra representation π_σ is fixed by the claim for $\sigma(\mathbf{x})$ field to have an expectation value

$$\phi_\sigma(\sigma^a(\mathbf{x})) = \phi_\sigma(\alpha^t[\sigma^a(\mathbf{x})]) = \sigma^a$$

not changing in time. For $\mathbf{R}^3$- translational invariant states σ^a must be constant.

Let us introduce $\mathcal{S}$ the minimal set of primar states over $\mathcal{A}_l$ containing $\phi_{\sigma\hat{\eta}}$ ($\sigma = \sigma\hat{\eta}$) and stable under $(\alpha^t_\pi)^*$, $(\beta^\varsigma)^*$. Denote $\Pi_\mathcal{S}$ the representation of $\mathcal{A}_l$ given by direct sum of all $\mathcal{S}$'s representations and $\mathcal{Z}_\mathcal{S}$ will be the center of $\Pi_\mathcal{S}(\mathcal{A}_l'')$. Here

$$\mathcal{S} = \{\phi_{\sigma_\infty(\hat{\mathbf{x}})}\,,\ \forall\ \sigma_\infty(\hat{\mathbf{x}})\}$$

with an arbitrary Lie-G valued function $\sigma_\infty(\hat{\mathbf{x}})$. Algebra $\mathcal{A}_l$ is implied to be weakly asymptotically abelian with respect to translations

$$w\text{-}\lim_{x\to\infty} \pi(\,[\,A_\mathbf{x}\ ,\ B\,]\,) = 0\,, \qquad A_x \equiv \alpha^\mathbf{x}[A] \quad \forall\ A,\ B \in \mathcal{A}_l\,. \tag{37}$$

Then the variables at infinity

$$\sigma^a_\infty(\hat{\mathbf{x}}) = w\text{-}\lim_{R\to\infty}\int_0^\infty dy\, f_R{}'\,\sigma^a(y\hat{\mathbf{x}})$$

exist in a weak topology of $\Pi_\mathcal{S}$ under the translation invariance of state ϕ:

$$\pi(\sigma_\infty) = \lim_{R\to\infty}\phi(\alpha_{R\hat{\mathbf{x}}}[\sigma]) = \phi(\sigma) = \sigma \tag{38}$$

$$[\sigma^a_\infty(\hat{\mathbf{x}})\ ,\ A] = w\text{-}\lim_{R\to\infty}\int_0^\infty dy\, f'_R(y)\int_0^\infty \frac{z^2\,dz}{\max(y,z)}\,[\,\Phi^a(z\hat{\mathbf{x}})\ ,\ A\,] = 0\,. \quad (39)$$

The last is true in virtue of finite integration region owing to (37) and supp $f_R{}' \in [R', R'']$. Hence we have that $\sigma_\infty(\hat{\mathbf{x}}) \in \mathcal{Z}_S$. The symmetry β^ς is commuting with the Hamiltonian and acts on variables at infinity by Ad^*-manner

$$-\,i\,[\,\Phi(\varsigma)\ ,\ \sigma^a_\infty\,] = gt^{abc}\,\varsigma^b(\hat{\mathbf{x}})\,\sigma^c(\hat{\mathbf{x}})\,. \quad (40)$$

The representation dependent effectively localized dynamics is

$$\dot{A}^a_k = E^a_k - P_{kl}\nabla_l(\,\sigma - \pi(\sigma_\infty)\,)^a\,, \quad (41)$$

$$\dot{E}^a_k = gt^{abc}\,E^b_k\,(\,\sigma - \pi(\sigma_\infty)\,)^c - P_{kl}\nabla_i F^a_{il}\,, \quad (42)$$

$$\dot{\sigma}^a(\mathbf{x}) = \frac{g}{2}t^{abc}\int_x^\infty \frac{dy}{y^2}\int_0^y z^2\,dz\,[\,\Phi^b(z\hat{\mathbf{x}})\ ,\ \sigma^c(z\hat{\mathbf{x}}) - \pi(\sigma_\infty)^c\,]_+ + \\ + \int_x^\infty dy\,\hat{x}_j\nabla_i\,F^a_{ij}(y\hat{\mathbf{x}})\,. \quad (43)$$

It is possible to derive from (43) the dynamics α^t_π of variables at infinity itself in the Hartree–Fock approximation[1] [18]

$$\dot{\sigma}^a_\infty(\hat{\mathbf{x}}) = -gt^{abc}\,\sigma^b_\infty(\hat{\mathbf{x}})\,\sigma^c\,,\quad \sigma^a_\infty(\hat{\mathbf{x}}) \in \mathcal{Z}_S \quad (44)$$

which is a classical rotations around $\hat{\eta}$ with frequency σ equation.

In the $\pi_{\sigma\hat{\eta}}$ representation a part of symmetry transformations β^ς is spontaneously broken. It accompanies by a mass gap generation in accordance with the generalized Goldstone theorem [17]. The mass gap coincides with σ.

4. PARTITION FUNCTION AND COLLECTIVE FIELDS

Let us exploit the path integral technique applying it to the partition function calculations. The Faddeev–Popov integral representation of the generating functional of correlation functions will be the starting point of them

$$Z[\zeta,\eta] \;=\; \int_{\leftrightarrow} \mathcal{D}\mathbf{A}_\perp\mathcal{D}\mathbf{E}_\perp\ \exp\,(-)\int_\Omega dx\,[\,\mathbf{E}\dot{\mathbf{A}} - \frac{1}{2}\mathbf{E}^2_{FS} + \frac{1}{2}\mathbf{B}^2_{FS} + \\ +\quad i\,\zeta E_\parallel - \eta B_\parallel\,]\,. \quad (45)$$

[1] which is a consequence of some clusterness conditions.

tions for fields at $t \in [0,\beta]$. It should be stressed that the infrared cut–off R is implied in Ω for the precaution aim against infrared singularities. Where

$$B^a_{FS\ k} = \frac{1}{2}\varepsilon_{kij}\, F^a_{ij}\,, \qquad \mathbf{E}_{FS} = \mathbf{E} + \hat{\mathbf{x}}\, E_{\|}(\mathbf{A},\mathbf{E})$$

with the Gauss law solution (24) substituted in. The integration in (45) is carried out over real transversal $\mathbf{A}$, whose derivatives as well as purely imaginary transversal $\mathbf{E}$ presented above are Lesbegue x-integrated with a square, i. e. they belong to L^2 space. The sources for the longitudinal components were introduced and they won't be written afterwards.

One simple observation is obvious enough. Writing

$$\mathbf{E}^2_{FS} = \mathbf{E}^2_{\perp} + E^2_{\|}\,, \qquad \mathbf{B}^2_{FS} = \mathbf{B}^2_{\perp} + B^2_{\|}\,, \qquad \mathbf{B}_{\perp} = P\,\mathbf{B}_{FS} = P\,rot\,\mathbf{A}$$

we can see, that the nonlocal and nonabelian counterparts of $\mathbf{E}_{FS}, \mathbf{B}_{FS}$ are combined in the longitudinal modes only. The last fact is a key idea of "dynamics diagonalization" method. Let us introduce two auxiliary real t-periodic variables $\lambda,\ \nu$ via the relations

$$\exp\,(\frac{1}{2}\int_{\Omega} dx\, E^2_{\|}\,) = \int_{\leftrightarrow} \mathcal{D}\lambda\ \exp\,(\int_{\Omega} dx\,[-\frac{1}{2}\lambda^2 + \lambda E_{\|}\,]\,)\,, \tag{46}$$

$$\exp\,(-\frac{1}{2}\int_{\Omega} dx\, B^2_{\|}\,) = \int_{\leftrightarrow} \mathcal{D}\,\nu\ \exp\,(\int_{\Omega} dx\,[-\frac{1}{2}\nu^2 + i\,\nu B_{\|}\,]\,)\,. \tag{47}$$

These two Gaussian integrals are calculated over $\lambda,\ \nu$ from L^2. Then we have

$$\int_{K_R} d\mathbf{x}\,\lambda(x\hat{\mathbf{x}})\frac{1}{x^2}\int_0^x y^2\,dy\,\Phi(y\hat{\mathbf{x}}) = \int_{K_R} d\mathbf{x}\,\Phi(\mathbf{x})\,\tilde{\sigma}_R(\mathbf{x})\,, \tag{48}$$

$$\tilde{\sigma}_R(\mathbf{x}) = \int_x^R dy\,\lambda(y\hat{\mathbf{x}}) \tag{49}$$

and perform integrations by parts

$$\int_{K_R} d\mathbf{x}\ \tilde{\sigma}_R(\mathbf{x})\,\partial\mathbf{E}_{\perp} = -\int_{K_R} d\mathbf{x}\ \mathbf{E}_{\perp}\,P\partial\,\tilde{\sigma}_R + R^2\int_{S_R} d\hat{\mathbf{x}}\ \mathbf{E}_{\perp}(R\hat{\mathbf{x}})\,\tilde{\sigma}_R(R\hat{\mathbf{x}})\,,$$

$$\int_{K_R} d\mathbf{x}\ \nu\,rot\,\mathbf{A}_{\perp} = \int_{K_R} d\mathbf{x}\ \mathbf{A}_{\perp}\,[\,\partial\,\nu\ ,\ \hat{\mathbf{x}}\,] - R^2\int_{S_R} d\hat{\mathbf{x}}\ \nu\ [\,\hat{\mathbf{x}}\ ,\ \mathbf{A}_{\perp}\,]\,.$$

The boundary terms are vanishing due to FS gauge and spherically symmetrical infrared regularization.

$$I = \int_{\leftrightarrow} \mathcal{D}\mathbf{E}_\perp \exp\left(\int_\Omega dx \left[\frac{1}{2}(\mathbf{E}^a)^2 - \mathbf{E}^a \left(\dot{\mathbf{A}}^a + P\partial\,\sigma^a - gt^{abc}\,\mathbf{A}^b\,\sigma^c \right) \right] \right) =$$

$$= \exp\left(-\frac{1}{2} \int_\Omega dx \left[\dot{\mathbf{A}}^a + P\partial\,\sigma^a - gt^{abc}\,\mathbf{A}^b\,\sigma^c \right]^2 \right). \tag{50}$$

Looking throughout these operations it is easy to notice that λ enter inside formulae thereby σ_R. We wish to make a natural change of the integration variable

$$\int \mathcal{D}\lambda \quad \to \int \mathcal{D}\,\tilde{\sigma}_R\,.$$

How much is it harmless ? Under retained cut–off there are no doubts in its allowance. One concludes from operatorial formulation that the infrared removal procedure evokes the variables at infinity appearance in the equations of motion. These variables are in fact order parameters characterizing the Hilbert space. So, we shall allow constant asymptotics of $w\text{-}\lim_{R\to\infty} \tilde{\sigma}_R = \sigma - \sigma_\infty$ variable.

It is instructive to perform rotation in present integral over $\mathbf{A}$, λ, ν

$$A_{\perp\,i} = \varepsilon_{ijk}\,\mathcal{A}_{\perp\,j}\hat{x}_k$$

preserving the integration region and with Jacobian equals to nonzero normalization constant. After we have made out necessary treatment let's write down the result

$$Z = \int_{\leftrightarrow} \mathcal{D}\,\mathcal{A}_\perp \mathcal{D}\,\sigma\,\mathcal{D}\,\nu\; exp\left(-\frac{1}{2}\nu^2 + \frac{1}{2}\sigma\,\Delta\,\sigma - \frac{1}{2}\mathcal{A}\bullet M \bullet \mathcal{A} + \mathcal{A}\bullet N \right), \tag{51}$$

$$M^{ab} = \left(-\Delta_x\,\delta^{ab} - (\nabla_t^2)^{ab} \right) P + i\,gt^{abc}\,\nu^c\,L\,, \tag{52}$$

$$N_i^a = i\,P_{ij}\,\partial_j\,\nu^a - L_{ij}\,\nabla_t^{ab}\,\partial_j\,\sigma^b\,, \qquad \nabla_t^{ab} = \delta^{ab}\,\partial_t - gt^{abc}\,\sigma^c\,. \tag{53}$$

The functional notations are more suitable since this moment. The integration $\int_0^\beta dt \int d\mathbf{x}$ is omitted everywhere and a bullet specifies a scalar product.

The integral over $\mathcal{A}$ may be evaluated straightforwardly and then using Appendix D formulae the vector structure can be eliminated at all. We shall achieve here just the same result in an easier way. One could rewrite (51) through $\mathcal{A}_\pm$ projections

$$Z = \int_{\leftrightarrow} \mathcal{D}\,\mathcal{A}_+ \mathcal{D}\,\mathcal{A}_- \mathcal{D}\,\sigma\,\mathcal{D}\,\nu\; \exp\left(-W[\sigma,\nu,\mathcal{A}_\pm] \right), \tag{54}$$

$$W[\sigma,\nu,\mathcal{A}_\pm] = \frac{1}{2}\nu^2 - \frac{1}{2}\sigma\,\Delta\,\sigma + \frac{1}{2}\mathcal{A}\bullet\mathcal{C}\bullet\mathcal{A} + i\,\mathcal{A}\bullet\mathcal{K}\,,$$

$$\mathcal{C} = \begin{pmatrix} 0 & C_+ \\ C_- & 0 \end{pmatrix}, \qquad D^{ab} = g t^{abc} \nu^c ,$$

$$C_\pm = -\Delta_x - \nabla_t^2 \pm D , \qquad K_\pm = \partial_\pm \nu \pm \nabla_t \partial_\pm \sigma .$$

The $\mathcal{A}_\pm$ integration is now a trifle. Hence we have

$$Z = \int_\leftrightarrow \mathcal{D}\sigma \mathcal{D}\nu \; \exp(-W[\sigma,\nu]) ,$$

$$\begin{aligned} W[\sigma,\nu] \;&=\; \frac{1}{2}\nu^2 - \frac{1}{2}\sigma\,\Delta\,\sigma + \frac{1}{2}K_- \bullet C_+^{-1} \bullet K_+ + \frac{1}{2}K_+ \bullet C_-^{-1} \bullet K_- + \\ &+ \;\; \frac{1}{2} tr\, log\, C_+ C_- . \end{aligned} \tag{55}$$

There are some remarks to be made.

First of all, to include an exterior source J of a gauge field A it is enough to add a term $\Delta W = gJ_0\sigma$ and change $K_\pm \to K_\pm \pm gJ_\pm$ in the previous formula.

Secondly, formula (55) is, as a matter of fact, an exact result and completely equivalent to the conventional formulation of QCD.

In the third, one could connect the generating functionals of $E_\parallel$, $B_\parallel$ (45) with

$$\mathcal{Z}[\zeta,\eta] = \int_\leftrightarrow \mathcal{D}\lambda\, \mathcal{D}\nu \; \exp\left[-W[\lambda,\eta] - i(\lambda\zeta + \eta\nu)\right] \tag{56}$$

by the relation proved in Appendix D

$$Z[\zeta,\eta] = \exp\left[\frac{1}{2}\int_\Omega dx\, (\zeta^2 + \eta^2)\right] \mathcal{Z}[\zeta,\eta] . \tag{57}$$

Let $F(E_\parallel, B_\parallel)$ be an arbitrary functional of the longitudinal strengths. Then its average

$$\langle F(E_\parallel, B_\parallel) \rangle = Z^{-1}[\zeta,\eta]\, F(i\frac{\delta}{\delta\zeta}, \frac{\delta}{\delta\eta})\; Z[\zeta,\eta]\Big|_{\zeta=\eta=0} \tag{58}$$

is connected with $\langle F(\lambda,\nu)\rangle$ of $\mathcal{Z}[\zeta,\eta]$'s by the formula

$$\begin{aligned} \langle F(E_\parallel, B_\parallel) \rangle = &\textstyle\int_\leftrightarrow \mathcal{D}\xi\, \mathcal{D}\theta\; \exp(-\frac{1}{2}\int_\Omega dx\,[\xi^2 + \theta^2]) \\ &\langle F(\lambda + i\xi, -i\nu + \theta) \rangle . \end{aligned} \tag{59}$$

could be imagined as longitudinal (chromo) electric and magnetic fields smoothed by Gaussian heat noise. Besides it is interesting that

$$\langle E_{\|} \rangle = \langle \lambda \rangle , \qquad \langle B_{\|} \rangle = -i \langle \nu \rangle , \tag{60}$$

$$\langle E_{\|} \rangle^* = -\langle E_{\|} \rangle , \qquad \langle B_{\|} \rangle^* = \langle B_{\|} \rangle . \tag{61}$$

Thus, although λ , ν are real in the integral, their expectations are purely imaginary.

5. RENORMALIZATION AS AN INTEGRATION PROCEDURE OVER FAST VARIABLES

At once one should investigate the effective action $W[\sigma,\nu]$ at a "classical" level, in spite of quantum contributions created by $\mathcal{A}$'s integration W may be named classical only in conditional sense of a word. Strictly speaking, the loops terminology is used here in accordance with the method of the saddle point. It maintains that the integration contour in σ and ν complex planes must be deformed in a way of the steepest descent of $-W[\sigma,\nu]$. Postponing such questions till the following section we pursue the aim to evaluate $W^{(1)}$ correction term. This task is urgent for renormalization in the effective action. The operators $C_\pm$ do not include angular differentiations thus tr $\log C_\pm$ will be inevitably ultra violet singular due to $\delta(\hat{\mathbf{0}}) = \frac{1}{4\pi}\sum_{l\geq o}(2l+1)$ factor. It is useful to realize how it disappears under corrections taken into account and how $\mathbf{R}^3$-covariance restores.

In the whole volume the σ,ν determinant computation is a hopeless problem. Let us implement a kind of an adiabatic approximation. We decompose Fourier variables into slow and fast parts

$$\sigma = \bar{\sigma} + \tilde{\sigma} , \qquad \begin{aligned} \bar{\sigma}(\mathbf{k}) &= \sigma(\mathbf{k})\,\theta(q - \mid \mathbf{k} \mid) , \\ \tilde{\sigma}(\mathbf{k}) &= \sigma(\mathbf{k})\,\theta(\mid \mathbf{k} \mid - q) \end{aligned}$$

and the same for ν. Then we neglect fast modes inside slowly changing functions, i. e. inside the ∇_t covariant derivatives, whereas $\bar{\sigma},\bar{\nu}$ are regarded there as constants

$$W[\sigma,\nu] = \bar{W}[\bar{\sigma},\bar{\nu}] + \tilde{W}[\tilde{\sigma},\tilde{\nu};\bar{\sigma},\bar{\nu}] .$$

residue. Tildes will be missed henceforth to simplify formulae.

Thus $\bar{\sigma}, \bar{\nu}$ are treated as constants inside $\bar{C}_{\pm}^{-1}$, $\bar{\nabla}_t$,then $\hat{\partial}_i$ is commuting with these operators. We have

$$\begin{aligned} W_2 &= \frac{1}{2}\nu^2 - \frac{1}{2}\sigma\,\Delta\,\sigma - \frac{1}{2}\nu\,x^{-1}\hat{\Delta}\left(\frac{\bar{C}_{+}^{-1}+\bar{C}_{-}^{-1}}{2}\right)x^{-1}\,\nu - \\ &- \frac{1}{2}\sigma\,x^{-1}\bar{\nabla}_t\hat{\Delta}\left(\frac{\bar{C}_{+}^{-1}+\bar{C}_{-}^{-1}}{2}\right)\bar{\nabla}_t x^{-1}\sigma - \frac{1}{2}\nu\,x^{-1}\hat{\Delta}\left(\frac{\bar{C}_{+}^{-1}-\bar{C}_{-}^{-1}}{2}\right)\bar{\nabla}_t x^{-1}\sigma \\ &- \frac{1}{2}\sigma\,x^{-1}\bar{\nabla}_t\hat{\Delta}\left(\frac{\bar{C}_{+}^{-1}-\bar{C}_{-}^{-1}}{2}\right)x^{-1}\,\nu\,. \end{aligned} \tag{62}$$

The integral over fast components modifies under the variables transformation

$$\begin{aligned} \sigma &= x\bar{\nabla}_t^{-1}\bar{C}_{-}\sigma'\,, \\ \nu &= x\bar{C}_{-}\nu'\,, \end{aligned} \qquad \frac{D(\sigma,\nu)}{D(\sigma',\nu')} = det\begin{pmatrix} x\bar{\nabla}_t^{-1}\bar{C}_{-} & 0 \\ 0 & x\bar{C}_{-} \end{pmatrix}.$$

In new variables the squared part of the exponential becomes

$$W_2 = \frac{1}{2}\begin{pmatrix}\nu' & \sigma'\end{pmatrix}\bullet\begin{pmatrix} \bar{C}_{+}x^2\bar{C}_{-} - \hat{\Delta}\bar{B} & \hat{\Delta}\bar{D} \\ -\hat{\Delta}\bar{D} & -\bar{C}_{+}\bar{\nabla}_t^{-1}x\,\Delta\,x\bar{\nabla}_t^{-1}\bar{C}_{-} + \hat{\Delta}\bar{B} \end{pmatrix}\bullet\begin{pmatrix}\nu' \\ \sigma'\end{pmatrix}.$$

The integral over fast fields may be calculated in the case $\bar{\nu} = 0$

$$\begin{aligned} W &= W^{(0)} + W^{(1)} = \frac{1}{2}\mathrm{tr}\,\log\left[\,(-\Delta + (\,\bar{\sigma} + i\,\partial_t\,)^2\,)\,(-\Delta + (\,\bar{\sigma} - i\,\partial_t\,)^2\,)\,\right] + \\ &+ \frac{1}{2}\mathrm{tr}\,\log\left[\,(-\mathcal{D} + (\,\bar{\sigma} + i\,\partial_t\,)^2\,)(-\mathcal{D} + (\,\bar{\sigma} - i\,\partial_t\,)^2\,)\,\right], \end{aligned} \tag{63}$$

$$\mathcal{D} = \Delta_x\,(1 + x^{-1}\,\Delta_x^{-1}\,x^{-1}\,\hat{\Delta}\,).$$

Hence the main $\delta(\hat{0})$-obstacle in the renormalization procedure has been overcame. This result does not depend upon that we have ignored $\bar{\nu}$ presence.

6. FREE ENERGY OF SU(2) YANG–MILLS THEORY

We calculate effective action (55) on constant ν, σ fields. In accordance with (60) it is necessary to substitute a purely imaginary magnetic field in tities $g\nu \to i\nu$, $g\sigma \to \sigma$. The last rescaling of fields was made for the sake of convenience. Employing the spectral decompositions for ∂_t, Δ_x operators one has derived the following result

$$\begin{aligned} W[\sigma,\nu] &= -\frac{\beta}{2g^2}(2\pi\delta(0))^3\nu^2 + 2\pi\delta(\hat{0})\,\delta(0)\int_0^\infty dk \sum_{n=-\infty}^{\infty} \mathrm{tr}\log\left(\left[(k^2+\omega^2)\,1 + \right.\right. \\ &+ \left.\left. i\,(2\omega\,\sigma+\nu)\,\mathcal{L} + \sigma^2\mathcal{P} \right) \right] [\nu \to -\nu] \,), \qquad (\omega = 2\pi n/\beta), \qquad (64) \end{aligned}$$

where $(2\pi\delta(0))^3$ is the full space volume and $\delta(\hat{0})$ is the ultraviolet divergent factor discussed above. It's worth noticing that we restrict ourselves by magnetic field ν^a of the same direction as σ^a in c-space. If transversal with respect to σ^a components of ν^a-field are counted, the operator inside log 's eigenvalues become nondegenerate. Nevertheless, the picture would not change drastically and it is easy to consider a more general case.

Let us introduce new notations for the free energy density $\mathcal{F}$

$$\mathcal{F} = W/[(2\pi\delta(0))^3\beta], \qquad \mathcal{F} = \frac{\delta(\hat{0})}{\beta\,\delta(0)^2}\,F, \qquad F = -a\,u^4 + \mathcal{U}[s,u] \qquad (65)$$

and for parameters

$$u = \frac{\beta}{2\pi}\sqrt{\nu}, \qquad s = \frac{\beta\,\sigma}{2\pi}, \qquad a = (2\alpha_s\beta^2\Lambda^2)^{-1}, \qquad \Lambda = (2\pi\,\delta(0))^{-1},$$

$$\alpha_s = \frac{\bar{g}^2}{4\pi}, \qquad \bar{g} = \frac{L}{2\pi}g, \qquad \delta(\hat{0}) = \frac{1}{4\pi}\sum_{l\geq 0}(2l+1) = \frac{L^2}{4\pi}.$$

Here $\bar{g}$ is treated as a renormalized coupling constant. It follows from (64) after subtraction of a trivial divergence

$$\begin{aligned} \mathcal{U}[s,u] &= \frac{1}{2\pi^2}\int_0^\infty dv \,\log\Big[\left(\cosh\sqrt{v^2+(2\pi u)^2} - \cos[2\pi s] \right) \\ &\quad \left(\cosh\sqrt{v^2-(2\pi u)^2} - \cos[2\pi s] \right) \Big/ \cosh^2 v \Big], \qquad (66) \end{aligned}$$

$$\mathcal{U}[s,0] = -\frac{1}{2}(1-2s)^2, \qquad (0 \leq s \leq 1). \qquad (67)$$

The integral can be converted into series

$$\mathcal{V}[s,u] \;=\; \mathcal{U}[s,u] - \mathcal{U}[s,0],$$

$$\mathrm{Re}\,\mathcal{V}[s,u] \;=\; \sum_{k=-\infty}\Big([\sqrt{(k+s)^2+u^2} + \sqrt{(k+s)^2-u^2}_{+}$$

$$- \quad 2\,|\,k+s\,|\Big)\,, \tag{68}$$

$$\mathrm{Im}\,\mathcal{V}[s,u] \;=\; \sum_{k=-\infty}^{\infty} \sqrt{u^2-(k+s)^2}_{+}\,. \tag{69}$$

The Poisson summation of Appendix E helps to transform this into cylindric functions expansions

$$\mathrm{Re}\,\mathcal{V}[s,u] \;=\; 2s(s-1) + u\sum_{k=1}^{\infty}\frac{\cos[2\pi ks]}{k}\Big(Y_1(2\pi ku) - \frac{2}{\pi}K_1(2\pi ku)\Big)\,, \tag{70}$$

$$\mathrm{Im}\,\mathcal{V}[s,u] \;=\; \frac{\pi u^2}{2} + u\sum_{k=1}^{\infty}\frac{\cos[2\pi ks]}{k}J_1(2\pi ks)\,. \tag{71}$$

The asymptotic formulae can be obtained using asymptotical behavior of the Neumann and Bessel functions. They are expressed through the periodically continued by Hurwitz formula of Appendix E generalized zeta–function

$$\mathrm{Re}\,\mathcal{U}[s,u] \;=\; \sqrt{2u}\,[\,\zeta_H(-\frac{1}{2},u+s) + \zeta_H(-\frac{1}{2},u-s)\,]\,, \tag{72}$$

$$\mathrm{Im}\,\mathcal{U}[s,u] \;=\; \frac{\pi u^2}{2} + \sqrt{2u}\,[\,\zeta_H(-\frac{1}{2},u+s) + \zeta_H(-\frac{1}{2},u-s)\,]\,. \tag{73}$$

If parameter "a" is more than approximately two, there is only trivial phase present, after that deeper minima arise at $u = s = 0.5$. These minima at an integer u, $s = 0$ and at half–integer u, $s = 0.5$, arising under smaller a-parameter, are unstable because of a nonzero imaginary part of the free energy, although their depth increases like a squared root of u.

The integration contours of the saddle point method are shown on Fig. 5. The first of them corresponds to deconfined phase, the other path approaches the branch points $\pm\sqrt{\sigma}$. A contour can not intersect the cuts anyway. This is the mathematical reason why unstable minima are not important for an asymptotical estimation of the effective action.

At the final stage we encounter with some formal infrared infinities like $\delta(0)$, meanwhile the physical quantities constructed from bare parameters should certainly be quite definite. All kinds of naive thermodynamical limits are in fact rather dangerous. The delicate mathematical treatment of this business requires an extremely unconstructive C^* apparatus [16]. The dry essence of it reveals that different phases of the system may be obtained by appropriate thermodynamical limits. The state regarded here possesses with the generalization of the equilibrium KMS state. We have got the result, but up–to–date theoretical physics is not clever enough to deal with it under all the precautions of rigour. Another, ultraviolet inspired cut–of ideology is not up to this case. The quantum bifurcation appears [17] and the properties of the cut–off system are absolutely different from an infinite one. The system which we describe is not finite, however numbers alike $\delta(0)$ must be treated in a sense of ordinary values. The nonstandard analysis really stands behind our operations.

7. CONFINEMENT–DECONFINEMENT PHASE TRANSITION

The phase transition under (4) critical temperature develops like a deeper minimum of the free energy emergence, while $\mathcal{F}$ jumps through discontinuity (see Fig.4). Hence the first order phase transition we have faced with proceeds in an explosion manner. The disposal is such that there appears an average magnetic field of a value

$$\nu_c = \pi^2 T_c^2 \tag{74}$$

creating a nonzero quantum phase $s = 0.5$. The last phenomenon governs confinement.

In the framework of the algebraic long–range dynamics Yang–Mills system is described by a representation $(\mathcal{A}_l\,,\ \Pi\,,\ \alpha^t_\Pi\,)$ of $(\,\mathcal{M}\,,\,\mathcal{F}\,,\alpha^t\,)$-system, where Π is an effective localization algebra $\mathcal{A}_l$ representation in $\mathcal{H}_\Pi$ Hilbert space with automorphisms α^t_Π unitary representable in it. Every such representation is characterized by order parameter σ^a stipulating a minimum of the free energy $\mathcal{F}(\beta, \sigma^a)$. It yields the dependance of a representation $\Pi_{\sigma(\beta)}$ upon β and due to variables at infinity involvement in the dynamical equations, the Heisenberg equations themselves depend upon β.

In Π_σ representation center elements become c-numbers with the classical equation of motion

$$\dot{\sigma}^a_\infty(\hat{\mathbf{x}}) = \sigma\,\epsilon^{abc}\,\sigma^b_\infty(\hat{\mathbf{x}})\,\hat{\eta}^c\,, \qquad \sigma^a = \sigma\,\hat{\eta}^a \tag{75}$$

and the Hamiltonian

$$h = -g \int d\hat{\mathbf{x}}\, \sigma\, \sigma^a_\infty(\hat{\mathbf{x}})\, \hat{\eta}^a\,. \tag{76}$$

In the case $\sigma(\beta) = 0,\ \beta < \beta_c$ variables at infinity are fixed at initial values corresponding to representations $\Pi_{\sigma_\infty(\hat{\mathbf{x}})}$. Thus the system "lives" at Π_0 because $\sigma^a_\infty(\hat{\mathbf{x}}) = 0$ provides $\mathcal{F}(\beta, \sigma)$ minimum. Otherwise, after $\beta > \beta_c\ \ \sigma(\beta) \neq 0$, evolution (75) throws out from any subrepresentation $\Pi_{\sigma_\infty(\hat{\mathbf{x}})}$, excluding the special initial values choice. Really the system lives in a larger space

$$\mathcal{H}_\Pi = \int d\sigma_\infty(\hat{\mathbf{x}})\, \mathcal{H}_{\sigma_\infty(\hat{\mathbf{x}})}\,.$$

The KMS condition

$$\langle A\, \alpha^{i\beta}[B] \rangle_\beta = \langle BA \rangle_\beta \qquad \forall\ A, B \in \mathcal{A}_l$$

on equilibrium state at the inverse temperature β with respect to α^t_Π dynamics defines it completely [21, 16]. The theorem about central decomposition at infinity [15] permits to write

$$Z = \int_{\mathcal{Z}_S} d\sigma_\infty(\hat{\mathbf{x}})\ e^{-\beta h}\ \mathrm{Tr}_{\mathcal{H}_{\sigma_\infty(\hat{x})}}\, e^{-\beta H}\,, \tag{77}$$

where the integration is carried out over Liouville classical phase space measure of variables at infinity, which coincides with the Haar measure. In virtue of the Liouville theorem about evolution invariance of this measure, the average (77) is KMS invariant. We have obviously on the primar spaces with purely imaginary expectations of variables at infinity were being shown to minimize the free energy

$$\mathrm{Tr}_{\mathcal{H}_{\sigma_\infty(\hat{x})}}\ e^{-\beta H} = \mathrm{Tr}_{\mathcal{H}_0}\ (\, e^{i\beta\, \Phi(\sigma_\infty)}\ e^{-\beta H}\,) \tag{78}$$

when the fact $[\Phi(\sigma_\infty), H] = 0$ is borne in mind (real expectations correspond to nonzero chemical potential). Thus

$$Z = \int_{\mathcal{Z}_S} d\sigma_\infty(\hat{\mathbf{x}})\ e^{s \int d\hat{x}\, (\sigma_\infty \hat{\eta})}\ \mathrm{Tr}_{\mathcal{H}_0}\, e^{i\beta\, \Phi(\sigma_\infty)}\ e^{-\beta H}\,, \tag{79}$$
$$g\beta\, \sigma = s\,, \qquad (0 \leq s \leq 2\pi)\,.$$

[2]Poisson structure is defined on orbits of the coadjoint representation by means of the Lie-algebra structure constants [20].

should average over the "period", i. e. the Haar group G measure

$$\begin{aligned} Z &= N \int d\sigma_\infty \, e^{s\int d\hat{\mathbf{x}}(\sigma_\infty \hat{\eta})} \int d\sigma_\infty \, \mathrm{Tr}_{\mathcal{H}_0} \left(e^{i\beta\Phi(\sigma_\infty)} \, e^{-\beta H} \right) = \\ &= N' \, \mathrm{Tr}_{\mathcal{H}_0} \left(P_s \, e^{-\beta H} \right). \end{aligned} \tag{80}$$

Due to the Peter–Weyl theorem [22] the operator

$$P_s = \int_G d\sigma_\infty(\hat{\mathbf{x}}) \, e^{i\Phi(\sigma_\infty)}$$

acting in $\mathcal{H}_\Pi$ is the local residual gauge group G singlet projector.

Just the same can be written for any correlation function

$$\langle F \cdots G \rangle_\beta = Z_s^{-1} \, \mathrm{Tr}_{\mathcal{H}_s} \left(e^{-\beta H} \, F \cdots G \right), \tag{81}$$

$$Z_s = \mathrm{Tr}_{\mathcal{H}_s} \, e^{-\beta H}, \qquad \mathcal{H}_s = P_s \, \mathcal{H}_0 \, . \tag{82}$$

This is precisely our definition of confinement.

8. CONCLUSION

The variables at infinity evoke a discolouration mechanism which is possible due to long–range interactions. Physically speaking, the quantum system in any given point feels the dynamics at infinity. This dynamics is closely related with the residual gauge transformations commuting with the time evolution. In the phase transition instant $\sigma_\infty(\hat{\mathbf{x}})$ appears as a boundary condition function on the formatting bubble surface inside unstable "trivial" vacuum. The colour shimmer on a bubble edge makes the whole medium colourless when it expands under temperature tends to zero. Long–range correlations don't decrease fast enough to stop this process.

As we could see, to insert confinement phenomenon unfolding a more delicate and fundamental structure into quantum field apparatus, the usual theory needs modifying. The algebraic QFT gives an adequate formalism for such cases as in well known statistical mechanics models [21]. From practical viewpoint it requires to consider expectations of observables involved into dynamics. We have calculated the SU(2) Yang–Mills theory free energy in a functional way applying asymptotical methods. The key idea is to perform an appropriate integration variables change in the path integral, for the Gaussian integrals it is always allowed. We managed to find for nice variables in the mathematically well defined 3-d Fock–Schwinger gauge.

tion as confinement phase transition can be realized only due to the Morchio and Strocchi construction.

Parameter "a," as it can be shown [23], is related to the string tension χ in the Wilson loop obeyed the area law at the phase of confinement

$$a = \pi^2 T^2 \chi^{-1}, \qquad \chi = 2\pi^2 \alpha_s \Lambda^2 \tag{83}$$

and this result is exact for rectangular contour. The numerical prediction $\chi \simeq (430 MeV)^2$ is quite correct.

After this first step let us formulate the program of future investigation. We are going to explore the propagators and to seek for physically important characteristics of confined phase like condensates. It is instructive to understand the role of spontaneous symmetry breaking which accompanies confinement under finite temperatures. There exists a technical problem of handling projected correlation functions, which are of the most interest indeed. How to construct the physically sensible S-matrix theory with really observable hadronic asymptotical states is not clear yet.

We wish thank to E. E. Boos for useful discussions since the very beginning of this work.

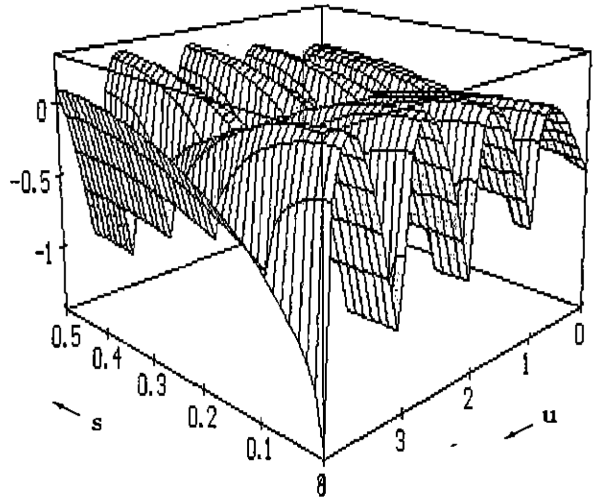

Fig. 1 Real part of the effective potential Re F dependance upon the parameters $0 \leq s \leq 0.5$ and $0 \leq u \leq 4$ under $a = 0.01$.

Im F

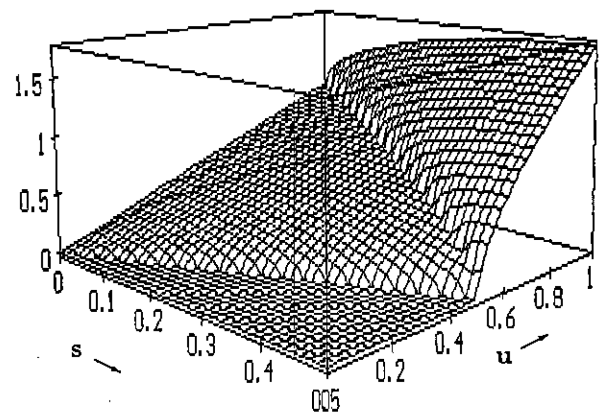

Fig. 2 Imagenary part of the effective potential Im F dependance upon $0 \leq s \leq 0.5$ and $0 \leq u \leq 1$.

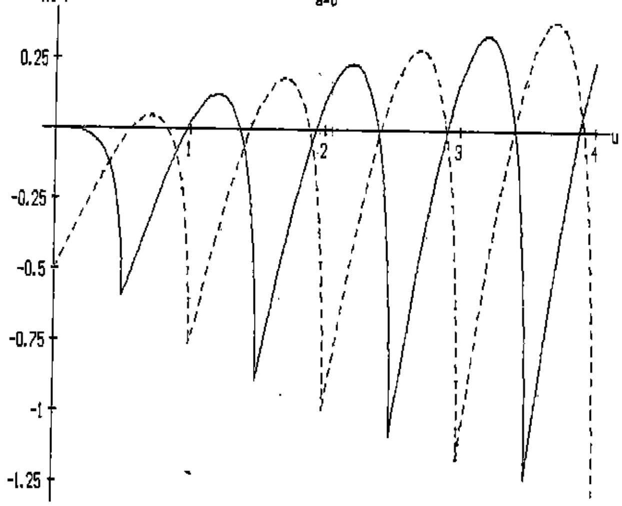

Fig. 3 Re F slices under $a = 0$ at fixed phases: solid line — $s = 0.5$, dashed line — $s = 0$.

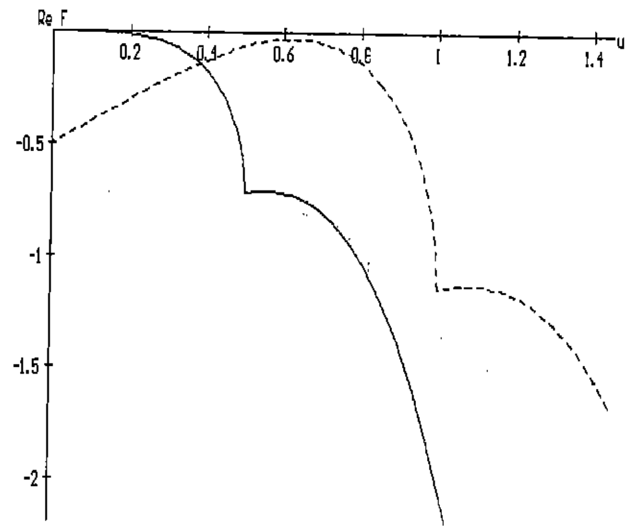

Fig. 4 Re F 's form at the instant of phase transition — solid line: $s = 0.5$, $a = 2$ and for nonstable minimum emergence — dashed line: $s = 0$, $a = 0.38$.

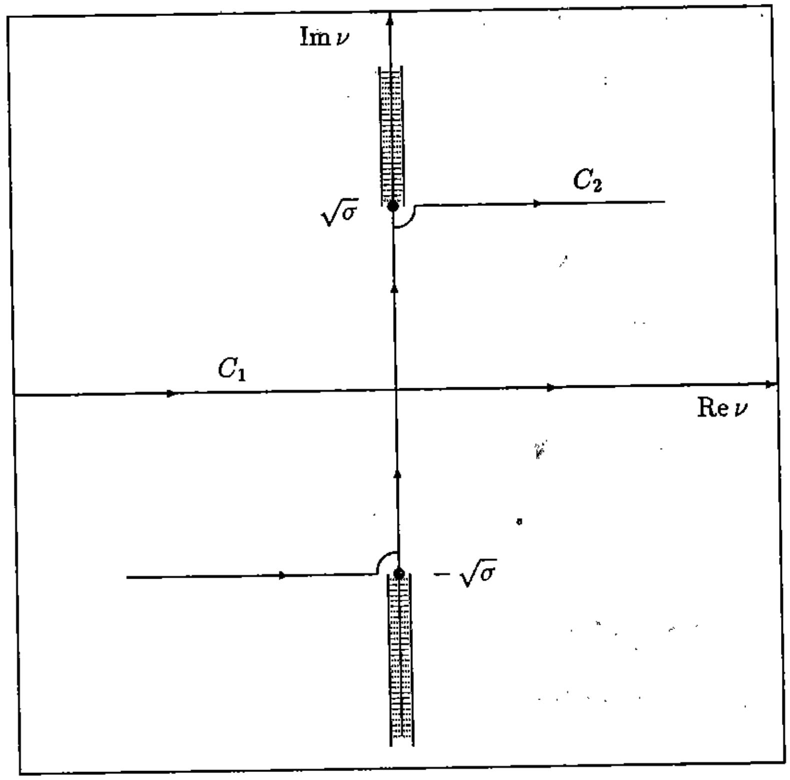

Fig. 5 Contours of the saddle point method: C_1 — trivial phase, C_2 — confined phase, where loose ends of the last should be deformed in the line of Im F = 0.

APPENDIX A
Transformation into FS gauge

Such transformation may be achieved by means of

$$\mathbf{A}_{FS}(\mathbf{x}) = U^{-1}\,(\,\mathbf{A}(\mathbf{x}) - g^{-1}\,\partial\,)\,U(\mathbf{x})\,, \qquad \mathbf{E}_{FS}(\mathbf{x}) = U^{-1}\mathbf{E}(\mathbf{x})\,U(\mathbf{x})\,,$$

where $U(\mathbf{x})$ is defined as the solution of this equation

$$\frac{\partial}{\partial\,x}U(\mathbf{x}) = g\hat{\mathbf{x}}\,\mathbf{A}(\mathbf{x})\,U(\mathbf{x})\,, \qquad U(\mathbf{0}) = \mathbf{1}\,.$$

Introducing Dyson P-exponential

$$U_\beta = P\,\exp\int_0^\beta d\alpha\,R(\alpha,\mathbf{x})\,, \qquad R^a(\alpha,\mathbf{x}) = g\,\mathbf{x}\mathbf{A}(\alpha\mathbf{x})$$

it is easy to derive

$$\begin{aligned}\mathbf{A}^b_{FS}(\mathbf{x}) \;&=\; \mathbf{A}^a(\mathbf{x})\,P\,\exp\int_0^1 d\alpha\,(-gt^{abc}\,R^c(\alpha,\mathbf{x})\,) - \\ &-\; \int_0^1 d\beta\,\partial R^a(\beta,\mathbf{x})\,P\,\exp\int_0^\beta d\gamma\,(-gt^{abc}\,R^c(\gamma,\mathbf{x})\,)\,.\end{aligned}$$

APPENDIX B
FS gluon propagator

Let's consider

$$D^{ij}_{FS}(x,y) = -i\,\langle 0\mid T_D\,(\,A^i_\perp(x)\,A^j_\perp(y)\,)\mid 0\rangle\,.$$

After reexpression through the Coulomb fields and using (18, 19) algebraic properties one gets eventually

$$\begin{aligned}D^{ij}_{FS}(\mathbf{x},\mathbf{y};x_0,y_0) \;&=\; -\int\frac{d^4k}{(2\pi)^4}\,\frac{e^{-i\,k_0\,(x_0-y_0)}}{(k^2+i\,0)}\Big[\delta^{ij}\,e^{i\,\mathbf{k}(\mathbf{x}-\mathbf{y})} - \frac{\partial}{\partial\,x_i}x^j\frac{e^{i\,\mathbf{kx}}-1}{i\,\mathbf{kx}} - \\ &-\; \frac{\partial}{\partial\,y_j}y^i\frac{e^{-i\,\mathbf{ky}}-1}{-i\,\mathbf{ky}} + \frac{\partial}{\partial\,x_i}x^l\frac{\partial}{\partial\,y_j}y^l\frac{e^{i\,\mathbf{kx}}-1}{i\,\mathbf{kx}}\frac{e^{-i\,\mathbf{ky}}-1}{-i\,\mathbf{ky}}\Big]\,.\end{aligned}$$

Owing to (2) boundary condition this propagator is well defined, whereas the axial gauge propagator has a nasty double pole term. Despite the axial gauge is very alike on our gauge, it suffers from serious problems with boundary conditions [3].

APPENDIX C

Connection between two generating functionals

To prove desirable (57) relation it is enough in (45) to present

$$e^{\int_\Omega dx\,(\eta B_\parallel - \frac{1}{2}\nu^2 + i\,\nu B_\parallel)} = e^{-\int_\Omega dx\,\frac{\nu^2}{2}}\, e^{-\int_\Omega dx\, i\,\eta \frac{\delta}{\delta\nu}}\, e^{\int_\Omega dx\, i\,\nu B_\parallel}$$

then integrate functionally by parts

$$\int_\leftrightarrow d\nu\,(\text{r.h.s. of above f.}) = \int_\leftrightarrow d\nu\, e^{-\int_\Omega dx\,(\frac{1}{2}[\nu + i\,\eta]^2 - i\,\nu B_\parallel)}$$

and in complete analogy for ν. From (57) the connection formula for any expectation values may be written after some computations with differential operators of translations in the functional space and using path representations for them.

APPENDIX D

Inverse matrix and determinant evaluations

Let P, L be operators with the algebra (7) and α, β, γ any operators commuting with them (not between each other). We shall denote here the whole trace by uppercase letter, the residual trace besides P, L 's degrees of freedom by lowercase one. Algebra allows to find

$$\begin{aligned} & (\alpha\, P + \beta\, L)^n = \alpha_n\, P + \beta_n\, L\,, \\ \alpha_n &= \mathrm{Re}\,(\alpha + i\,\beta)^n \equiv \frac{1}{2}[\,(\alpha + i\,\beta)^n + (\alpha - i\,\beta)^n\,]\,, \\ \beta_n &= \mathrm{Im}\,(\alpha + i\,\beta)^n \equiv \frac{1}{2\,i}[\,(\alpha + i\,\beta)^n - (\alpha - i\,\beta)^n\,]\,, \end{aligned}$$

which gives

$$\begin{aligned} \mathrm{Tr}\log[\,\gamma\,\mathbf{1} + \alpha\, P + \beta\, L\,] &= \mathrm{tr}\log\gamma + \mathrm{tr}\log[\,(\alpha + \gamma + i\,\beta)\,(\alpha + \gamma - i\,\beta)\,]\,, \\ (\,\gamma\,\mathbf{1} + \alpha\, P + \beta\, L\,)^{-1} &= \gamma^{-1}(1 - P) + \mathrm{Re}\,(\,\alpha + \gamma + i\,\beta)^{-1}\, P + \\ &+ \ \mathrm{Im}\,(\,\alpha + \gamma + i\,\beta)^{-1}\, L\,. \end{aligned}$$

APPENDIX E
Some special functions formulae

The Poisson summation

$$\sum_{k=-\infty}^{\infty} f(2\pi k) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} d\tau\, e^{-ik\tau} f(\tau)\,.$$

The cylindric functions integrals

$$\int_0^\infty dy\, \frac{\cos(ky)}{(y^2+v^2)^{\frac{1}{2}}} = \mathrm{K}_0(kv)\,, \qquad -\int_v^\infty dy\, \frac{\cos(ky)}{(y^2-v^2)^{\frac{1}{2}}} = \frac{\pi}{2}\mathrm{Y}_0(kv)\,,$$

$$\int_o^u v\, dv\, \mathrm{K}_0(kv) = \frac{1}{k^2}(-ku\,\mathrm{K}_1(ku)+1)\,,$$

$$\frac{\pi}{2}\int_0^u v\, dv\, \mathrm{Y}_0(kv) = \frac{1}{k^2}(\frac{\pi}{2}ku\,\mathrm{Y}_1(ku)+1)\,.$$

The Hurwitz representation of the generalized zeta–function [24], which is valid if $\mathrm{Re}\, s < 0$

$$\zeta(s,v) = 2(2\pi)^{s-1}\,\Gamma(1-s)\sum_{k=1}^{\infty} n^{s-1}\,\sin(2\pi n v + \pi s/2)\,.$$

We use the periodic continuation of zeta–function provided by this formula and denoted it as ζ_H. For any integer k

$$\zeta_H(-\frac{1}{2},k) = -\frac{\zeta(3/2)}{4\pi}\,, \qquad \zeta_H(-\frac{1}{2},k+\frac{1}{2}) = (1-1/\sqrt{2})\frac{\zeta(3/2)}{4\pi}\,.$$

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